Homework #3

Due: Mon., Feb. 11^{th}, 2019

#1 Convergence – By Definition Prove that the following sequences converge. Use the definition of convergence. Don't use any "fancy" theorems.

(a) $\left\{ \dfrac{5n}{n+1} \right\}_{n=1}^{\infty}$

(b) $\left\{ \dfrac{2n+1}{n^3+3n-1} \right\}_{n=1}^{\infty}$

#2 Convergence Redux Redo problem #1. This time show convergence using nothing more than the convergence of $\{1/n\}_{n=1}^{\infty}$ and some sequence arithmetic.

Example: $\dfrac{n^2+1}{3n^2-n+5} = \dfrac{n^2}{n^2} \cdot \dfrac{1+1/n^2}{3-1/n+5/n^2} \to \dfrac{1+0}{3-0+5\cdot 0} = \dfrac{1}{3}$ since $1/n \to 0$ implies $1/n^2 \to 0^2 = 0$ etc.

#3 Divergence – Subsequence Style $\left\{ \dfrac{(-1)^n n^2+2n+1}{3n^2-n+5} \right\}_{n=1}^{\infty}$ diverges. Show this using subsequences.

#4 Convergence Abstractly Let $\{a_n\}_{n=1}^{\infty}$ be a sequence which converges to L.

(a) Let $b_n = \dfrac{a_n + a_{n+1}}{2}$. Show that $b_n \to L$

(b) Suppose that $a_n \leq M$ for all $n \in \mathbb{N}$. Show that $L \leq M$.

#5 Proceed with Cauchy! Suppose that $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are Cauchy sequences. Show that $\{a_n+b_n\}_{n=1}^{\infty}$ is Cauchy.

(a) Do this using the definition of Cauchy sequences.

(b) Reprove this using Theorem 2.20 (i.e., a sequence is Cauchy if and only if it converges) and a theorem from Section 2.2. Cite the Theorem.

RESUBMIT Type up Homework #2 Problem #3 and its solution in LaTeX.