Math 3220 # Homework #5 **Due: Fri., Mar. 15^{th}, 2019**

#1 Defining Terms State the definition of $\lim_{x\to a^+} f(x) = -\infty$.

Adapt one of the "variant limit definitions" given in class. Make sure you include an appropriate assumption about the domain of f.

#2 Take it to the Limit Use the ϵ-δ definition of the limit (or one-sided limit) to prove the following:

(a) $\lim_{x\to 2} x^2 = 4$

(b) $\lim_{x\to 1^-} \sqrt{1-x} = 0$

#3 One More Time Let $f(x) = \begin{cases} 3x+1 & x \geq 1 \\ 2 & x < 1 \end{cases}$

Obviously, $\lim_{x\to 1} f(x)$ does not exist.

(a) Use (the negation of) the ϵ-δ definition of the limit to show that this limit does not exist.

(b) Give a quick argument that this limit does not exist using left and right handed limits.

#4 Squeezing In Another Limit Let f, g, and h be defined on $I - \{a\}$ where I is an open interval containing a. In addition, suppose that $f(x) \leq g(x) \leq h(x)$ for all $x \in I - \{a\}$. Finally, assume that $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$. Prove that $\lim_{x\to a} g(x) = L$.

Prove this using Lemma 3.2 (i.e., use the squeeze theorem for sequences to get this squeeze theorem).

#5 Linear Things are Continuous Let $A, B \in \mathbb{R}$ and define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = Ax + B$. Prove that f is continuous (everywhere). Do this using the ϵ-δ definition of continuity.

#6 Continuity Continued Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Suppose that $f(r) = 0$ for all $r \in \mathbb{Q}$ (f is zero on all of the rational numbers). Show that $f(x) = 0$ for all $x \in \mathbb{R}$.

RESUBMIT Type up Homework #4 Problem #3 and its solution in LaTeX.