Math 3220 # Homework #6 **Due: Wed., Apr. 3^{rd}, 2019**

#1 By the Definition Use the limit definition of the derivative to find the derivative of $f(x) = 3x^2 - x + 5$.

#2 Piecewise Let $f(x) = \begin{cases} x+1 & x < 2 \\ x^2 - 3x + 5 & x \geq 2 \end{cases}$

(a) Show that f is differentiable everywhere.
You can use the usual derivative rules for $x \neq 2$. For $x = 2$, use the limit definition and approach from the left and right separately.

(b) Suppose we change our function a little: $g(x) = \begin{cases} x+100 & x < 2 \\ x^2 - 3x + 5 & x \geq 2 \end{cases}$
Where is g differentiable? Explain your answer.

#3 Badly Behaved Let $f(x) = \begin{cases} 0 & x \in \mathbb{Q} \\ x^2 & x \in \mathbb{R} - \mathbb{Q} \end{cases}$

This is a lot like Dirichlet's function.

(a) Show that f is discontinuous at $x = a$ for all $a \neq 0$.
Suggestion: Let $a \neq 0$. Approach via a sequence of rationals and then via irrationals.

(b) Show that f is differentiable at $x = 0$. [You must use the limit definition of the derivative.]

(c) Where is $f(x)$ continuous? Where is $f(x)$ differentiable? Explain you answers using your results from parts (a) and (b).

#4 Easy Rule Let $f(x)$ be differentiable at $x = a$. Let k be some constant. Show that $(kf)'(a) = k\,f'(a)$ (i.e., we can pull constants out of derivatives).

RESUBMIT Type up Homework #5 Problem #5 and its solution in LaTeX.