Math 3220 # Homework #7 **Due: Fri., Apr. 12^{th}, 2019**

#1 Rolle's Rolle's Let f be twice differentiable on (a, b) and suppose that both f and f' are continuous on $[a, b]$. Suppose that $a < c < b$ and that $f(a) = f(c) = f(b)$. Prove that there exists some point in $a < x_0 < b$ such that $f''(x_0) = 0$.

Hint: What does $f(a) = f(b)$ say about f'?

#2 Contractions If we have a function $f : \mathbb{R} \to \mathbb{R}$ such that $|f(x) - f(y)| \leq |x - y|$ for all $x, y \in \mathbb{R}$, then we say that f is a *contraction*.

(a) Let f be a contraction. Show that f is uniformly continuous.

(b) Suppose that f is differentiable everywhere and that $|f'(x)| \leq 1$ for all $x \in \mathbb{R}$. Show that f is a contraction.
Suggestion: MVT.

#3 Bounded and Uniform Let $\{f_n\}$ be a sequence of functions defined on I and suppose that $\{f_n\}$ converges uniformly to f on I. In addition, assume that for each $n \in \mathbb{N}$, we have that f_n is bounded on I. Show that f must be bounded on I.

#4 Composition **[EXTRA CREDIT]** Let $\{f_n\}$ be a sequence of *continuous* functions on some interval I and suppose that $\{f_n\}$ converges uniformly to f on I. Show that given some sequence $\{x_n\} \subseteq I$ such that $x_n \to x \in I$ we have that the sequence $\{f_n(x_n)\}$ converges to $f(x)$.

RESUBMIT Type up Homework #6 Problem #4 and its solution in LaTeX.