Math 3220 # Homework #8 **Due: Mon., Apr. 29^{th}, 2019**

#1 Riemann Sums It might be helpful to know that: $\sum_{k=1}^{n} 1 = n$, $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, or $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

(a) Consider $I = \int_{-1}^{2} x^2\,dx$. Find the regular partition of $[-1,2]$ into $n = 6$ subintervals.
Compute Riemann sums using (i) left, (ii), right, and (iii) midpoint sample points.

(b) Briefly explain why $f(x) = x^2$ is integrable on $[-1,2]$. Find a general formula for a right hand (Riemann) sum approximation of $I = \int_{-1}^{2} x^2\,dx$ when using a regular partition of $[-1,2]$ into n subintervals.

(c) Compute your sum from part (b), then use that to compute I.
Confirm your result by using the Fundamental Theorem of Calculus.

#2 Integral Estimating Suppose that f is continuous on $[0,2]$, $f(0) = 5$, and $f'(x) > 0$ for all $0 < x < 2$.

Show that $\int_0^2 f \geq 10$.

#3 Squeezing Let g be integrable on $[a,b]$ with $\int_a^b g = 0$. Suppose that $0 \leq f(x) \leq g(x)$ for all $x \in [a,b]$.

Show that f is integrable on $[a,b]$ and that $\int_a^b f = 0$.

#4 Linear Let f and g be integrable on $[a,b]$.

Use the definition of the Riemann integral to show that $f+g$ is integrable with $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.

RESUBMIT Type up Homework #7 Problem #2 and its solution in LaTeX.