Math 3220 | # Final Revisions | **Due: Final Exam**

Homework #1 Problem #4 Let N be a positive integer and $x_1, \ldots, x_N \in \mathbb{R}$.

Show that $|x_1 + \cdots + x_N| \leq |x_1| + \cdots + |x_N|$. *Note:* Prove this using induction on N.

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Homework #2 Problem #3 Let $S \subseteq \mathbb{R}$ be non-empty and bounded below. Let $-S = \{-x \mid x \in S\}$. Show that $\sup(-S)$ exists. Then show that $-\inf(S) = \sup(-S)$.

This problem shows that the completeness axiom guaranteeing the existence of supremums implies a similar statement about the existence of infimums. Write down an "infimum" version of the completeness axiom.

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Homework #3 Problem #1 Prove that the following sequences converge. Use the definition of convergence. Don't use any "fancy" theorems.

(a) $\left\{ \dfrac{5n}{n+1} \right\}_{n=1}^{\infty}$

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(b) $\left\{ \dfrac{2n+1}{n^3+3n-1} \right\}_{n=1}^{\infty}$

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Homework #4 Problem #3 Let $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ be convergent series.

(a) Prove that $\sum_{k=1}^{\infty}(a_k + b_k)$ converges to $\sum_{k=1}^{\infty} a_k + \sum_{k=1}^{\infty} b_k$.

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(b) Suppose that $\sum_{k=1}^{\infty} c_k$ diverges. Can we conclude that $\sum_{k=1}^{\infty}(a_k + c_k)$ diverges as well? Prove this or give a counter-example.

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Homework #5 Problem #5 Let $A, B \in \mathbb{R}$ and define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = Ax + B$.

Prove that f is continuous (everywhere). Do this using the ϵ-δ definition of continuity.

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Homework #6 Problem #4 Let $f(x)$ be differentiable at $x = a$. Let k be some constant.

Show that $(kf)'(a) = k\,f'(a)$ (i.e., we can pull constants out of derivatives).

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Homework #7 Problem #2 If we have a function $f : \mathbb{R} \to \mathbb{R}$ such that $|f(x) - f(y)| \leq |x - y|$ for all $x, y \in \mathbb{R}$, then we say that f is a *contraction*.

(a) Let f be a contraction. Show that f is uniformly continuous.

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(b) Suppose that f is differentiable everywhere and that $|f'(x)| \leq 1$ for all $x \in \mathbb{R}$. Show that f is a contraction. *Suggestion:* MVT.

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Homework #8 Problem #4 Linear Let f and g be integrable on $[a,b]$.

Use the definition of the Riemann integral to show that $f+g$ is integrable with $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.

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