Math 3510 # Homework #1 **Due: Wed., Sept. 5^{th}, 2018**

#1 Jacobians! Consider $f(x,y) = (3y + x^2y + 1, -x + y^2 - 1, e^{xy^2})$.

(a) Find the Jacobian of f.

(b) Give the linear approximation of $f(x,y)$ based at $(x,y) = (0,-2)$

(c) Find something that could be called the "second derivative" of f (there are several ways to organize such a beast).

(d) Give the quadratic approximation of $f(x,y)$ based at $(x,y) = (0,-2)$.

#2 The Second Derivative Test: Let $f(x,y,z) = -2x^3 - 6xy^2 + 6x + z^4 - 2z^2$. Find and classify the critical points of f using the second derivative test.

Note: There are 12 critical points – sorry. Find them and tell me whether each point is a relative minimum, relative maximum, a saddle point, or if the test does not apply.

Strong Suggestion: Doing this problem by hand is not advised – maybe not even possible. You should use some software. Here are possibly relevant Maple commands or stubs of commands…

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with(LinearAlgebra):
with(VectorCalculus):
pts := solve({diff(???,?)=0, ... });
H := Hessian(???,[x,y,z]);
for pt in pts do
 Eigenvalues(subs(pt,H));
end do;
```

#3 There's got to be an easier way: Compute $\det \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ *only* using the fact that the determinant is multilinear, alternating/skew, and $\det(I_3) = 1$.

#4 Forms Basics: Let $\omega = xy\,dx + x^2z\,dz$, $\nu = xy^2z\,dx - xyz\,dz$, and $\eta = x^3\,dy\wedge dz + xy\,dz\wedge dx + yz\,dx\wedge dy$. In addition, let $\mathbf{p} = (1,2,-1)$, $\mathbf{a} = \langle 1,2,3\rangle$, and $\mathbf{b} = \langle -1,3,0\rangle$.

Note: Simplify answers in term of functions and forms: $dx, dy, dz, dy\wedge dz, dz\wedge dx, dx\wedge dy$, and $dx\wedge dy\wedge dz$.

(a) Evaluate $\omega_{\mathbf{p}}(\mathbf{a})$ and $\eta_{\mathbf{p}}(\mathbf{a},\mathbf{b})$.

(b) Compute $\omega\wedge\nu$ and $\omega\wedge\eta$.

(c) Compute $\eta\wedge\omega$ and $\omega\wedge\nu\wedge\eta$ (this part should be very easy).