Math 3510 # Homework #6 **Due: Fri., Nov. 9^{th}, 2018**

#1 The Unit Circle is a Manifold Let $M = S^1 = \{(x,y) \in \mathbb{R} \mid x^2+y^2=1\}$ (i.e., the unit circle).

(a) Show S^1 is a manifold using Theorem 3.2 (the cheaty/easy way).

(b) As we did with the unit sphere, define maps $\varphi_x^{\pm}$ and $\varphi_y^{\pm}$. For example: $\varphi_x^- : \{(x,y) \in S^1 \mid y < 0\} \to ???$ (you figure out the proper open subset of $\mathbb{R}$) is defined by $\varphi_x^-(x,y) = x$.

Show S^1 is a manifold with these maps forming an atlas.

Don't forget to explain why $\varphi_x^+, \varphi_x^- \varphi_y^+, \varphi_x^-$ are valid charts. Also, you should carefully write down transition functions (complete with domains/ranges) and explain why they are smooth.

(c) Consider $\tau^{-1} : (0, 2\pi) \to S^1 - \{(1,0)\}$ defined by $\tau^{-1}(\theta) = (\cos(\theta), \sin(\theta))$. Explain why τ is a possible chart. Then show that τ is compatible with your atlas of φ's defined in part (b).

(d) Given an atlas such that the determinants of all of the Jacobian's of transition functions are all always positive, we have an *oriented atlas* for our *oriented manifold*. Find such an oriented atlas for S^1.

(e) Show $f : S^1 \to \mathbb{R}$ defined by $f(x,y) = x^2 + 3xy$ is a smooth function using the atlas structure defined in part (b).

#2 The Torus is a Manifold We showed that the torus $T = \{(x,y,z) \mid (3-\sqrt{x^2+y^2})^2 + z^2 = 1\}$ is an embedded submanifold of $\mathbb{R}^3$ using Theorem 3.2 in class. The calculation was a bit involved. It is much easier to show that the torus $T = \{(x,y,z,t) \mid x^2+y^2 = 1 \text{ and } z^2+t^2=1\} = S^1 \times S^1$ is a embedded submanifold of $\mathbb{R}^4$ using Theorem 3.2. Show this.