Math 4010-101 # Homework #1 **Due: Tue., Jan. 21st, 2014**

1. Consider $G = \langle x, y, z \mid x^3 = 1, y^2 = 1, z = 1, xyxy = 1\rangle$. What is the order of G? Write down a Cayley table for G. What familiar group is G isomorphic to?

2. Prove $G = \langle x, y \mid xy = yx\rangle$ is an infinite group. [You must use the universal property.]

3. [Grad Problem] Let X_1 and X_2 be arbitrary sets (they could be empty, finite, or infinite). Prove that $|X_1| = |X_2|$ (i.e. there is a bijection between X_1 and X_2) implies that $F(X_1) \cong F(X_2)$.

   This means that sets of the same "size" yield isomorphic free groups. The converse of this statement is also true: If $F(X_1) \cong F(X_2)$ then $|X_1| = |X_2|$. However, this implication is a bit harder to prove (challenge problem).

   We know that subgroups of free groups are themselves free (this is the difficult to prove Nielsen-Schreier theorem). So if H is a subgroup of $F(X)$, then there is some set Y such that $H = F(Y)$. It is interesting to note that if $F(Y)$ is a subgroup of $F(X)$, then it is **not necessarily true** that $|Y| \leq |X|$!