Math 4010/5530-101 **Homework #3** **Due: Tue., Feb. 4^{th}, 2014**

1. [1.7 #16] Let G be a group. It is often useful to consider a group acting on itself. One important action of G on itself is *conjugation*. Define our action as follows (for all $g \in G$ and $x \in X = G$) $g \cdot x = gxg^{-1}$. Prove that this is in fact a group action. [*Note:* One can also define $x \cdot g = g^{-1}xg$. This yields a *right* group action. Many group theorists prefer to act on the right and thus define conjugation this way.]

2. [2.2 #5] Show that for the specified group G and subgroup H that $C_G(H) = H$ and $N_G(H) = G$.

   (a) $G = S_3$ and $H = \langle (123) \rangle = \{(1), (123), (132)\}$

   (b) $G = D_{2\cdot 4} = \langle r, s \mid r^4 = s^2 = rsrs = 1 \rangle$ and $H = \{1, s, r^2, r^2 s\}$

   (c) $G = D_{2\cdot 5} = \langle r, s \mid r^5 = s^2 = rsrs = 1 \rangle$ and $H = \langle r \rangle = \{1, r, r^2, r^3, r^4\}$

   Recall that $C_G(A) = \{g \in G \mid ga = ag \text{ for all } a \in A\}$ is the *centralizer* of A in G and $N_G(A) = \{g \in G \mid gAg^{-1} = A\}$ is the *normalizer* of A in G.

3. [2.3 #16] Let $x, y \in G$ and assume $|x| = n$ and $|y| = m$ (x and y have finite orders n and m respectively). Suppose that $xy = yx$ (they commute).

   (a) Prove that $|xy|$ divides $\text{lcm}(n, m) = \ell$.

   (b) Need this be true if x and y fail to commute?

   (c) Give an example (where x and y commute) such that $|xy|$ is not equal to the $\text{lcm}(n, m)$.

4. [**Grad Problem** 2.5 #11] Consider the group $QD_{16} = QD_{2\cdot 8} = \langle \sigma, \tau \mid \sigma^8 = \tau^2 = 1, \sigma\tau = \tau\sigma^3 \rangle$. This group is called the *quasidihedral* or *semidihedral* group of order 16. This group has three subgroups of order 8: $\langle \tau, \sigma^2 \rangle \cong D_{2\cdot 4}$, $\langle \sigma \rangle \cong \mathbb{Z}_8$, and $\langle \sigma^2, \sigma\tau \rangle \cong Q_8$ (the quaternions) and every proper subgroup is contained in one of these three subgroups.

   Fill in the missing subgroups in the lattice of all subgroups of the quasidihedral group (shown below). Exhibit each subgroup with no more than 2 generators. [This is an example of a nonplanar lattice.]