Math 4010/5160 # Homework #5 **Due: Wed., Mar. 21^{st}, 2018**

#1 Let's Get Integrating: First, parameterize the given curve. Then use that parameterization to compute the integral.

(a) Let C be the line segment from $A = 1 - i$ to $B = -2i$. $\displaystyle\int_C (3z^2 - 4z - 5)\,dz$

(b) Let C be the upper-half of the circle $|z+1| = 2$, oriented counter-clockwise. $\displaystyle\int_C \frac{z^2}{1+z}\,dz$

#2 Let's Estimate Use the ML-estimate to show that $\displaystyle\left|\oint_{|z-1|=1} \frac{e^z}{z+1}\,dz\right| \leq 2\pi e^2$.

#3 Fundamentally Flawed: Recall the fundamental theorem of calculus (for analytic functions) in Gamelin IV.2.

(a) Recompute #1(a) using the FTC for analytic functions.

(b) Let C be parameterized by $\gamma(t) = (\sin(2t) + t^2) + i(3\cos(t) + 1)$ where $-\pi \leq t \leq 0$.
Compute $\displaystyle\int_C (2z + \cos(z) + 6e^{3z})\,dz$.

#4 Watch Out! Work with Cauchy: Compute the following integrals using the Cauchy integral formula and its consequences.

(a) Compute $\displaystyle\oint_{|z|=2} \frac{2z-1}{z^2(z+3)}\,dz$ and $\displaystyle\oint_{|z-3|=2} \frac{2z-1}{z^2(z+3)}\,dz$.

(b) Compute $\displaystyle\oint_C \frac{dz}{z^2+4}$ where C is any counter-clockwise oriented closed loop which avoids $z = \pm 2i$. There are 4 distinct cases (consider when each of the points $z = \pm 2i$ are inside or outside your curve). Draw a representative curve in these 4 distinct situations and compute the value of this integral in each case.

(c) **[Grad. Problem]** $\displaystyle\int_0^{2\pi} \frac{1}{3 + \sin(\theta) + \cos(\theta)}\,d\theta$. *Note:* This problem is Fisher 2.3 #6. You should convert it back to a contour integral. Then apply the Cauchy integral formula. You might want to use technology for some of the algebraic manipulations and factoring the complex quadratic that arises.

#5 Seems Pretty Fundamental: [Grad. Problem] Prove that a complex polynomial with no zeros must be constant. Use Cauchy's Theorem and an ML-estimate. You may use the fact that $|P(z)| \geq c|z|$ when $|z|$ sufficiently large (c is some fixed, positive constant). Begin by supposing $P(z)$ is a non-constant polynomial then write $P(z) = P(0) + zQ(z)$. Divide this by $zP(z)$ to get $\displaystyle\frac{1}{z} = \frac{P(0)}{zP(z)} + \frac{Q(z)}{P(z)}$. Integrate around a circle of radius R and see what happens as $R \to \infty$. Contradiction?