#1 Bumping Up Against Infinity Find the **radius of convergence** of the power series for the following functions if they were expanded about the given point. [This is Gamelin V.4 #1a,b,c]

(a) $\dfrac{1}{z-1}$ about $z = i$ 　　　　(b) $\sec(z) = \dfrac{1}{\cos(z)}$ about $z = 0$ 　　　　(c) $\operatorname{sech}(z) = \dfrac{1}{\cosh(z)}$ about $z = 0$

#2 Standard Series Manipulation Find the power series expansion of the principal branch of $\arctan(z)$ expanded about $z = 0$. What is the radius of convergence of this series? *Hint:* Find an expansion for the derivative of arctan using a geometric series and then integrate it. [This is Gamelin V.4 #7]

#3 Infinite Power Find the power series expansion about $z = \infty$ for $\dfrac{z^2}{z^3 - 1}$. Where does this series converge? [This is part of Gamelin V.5 #1]

#4 Who Needs Polynomial Long Division? Calculate up to (and including) the fifth order terms in the power series expansion about $z = 0$ (where do these expansions converge?): [This is parts of Gamelin V.6 #2 and #3]

$$\text{(a)}\quad \frac{z}{\sin(z)} \qquad\qquad \text{(b)}\quad \frac{e^z}{1+z}$$

#5 Establishing Order For each of the following functions, first, find the zeros and their orders. Next, determine if they are analytic at ∞ and determine orders of any zeros there. [This is Gamelin V.7 #1d,f and #2d,f]

$$\text{(a)}\quad \cos(z) - 1 \qquad\qquad \text{(b)}\quad \frac{\cos(z)-1}{z}$$

#6 This is Easy – Don't Make it Hard Show that all of the zeros of $\sin(z)$ and $\tan(z)$ are simple. [This is Gamelin V.7 #3]

#7 Laurent Has Decomposed Consider $f(z) = \dfrac{1}{z^2 - z}$ [This is Gamelin VI.1 #1a and #2a]

(a) Find all of the Laurent expansions centered at $z = 0$.

(b) Find the Laurent expansion centered at $z = -1$ that converges at $z = 1/2$. Determine the largest open set where this expansion converges.

#8 I am Just a Simple Pole in a Complex Plane Find the isolated singularities of the following functions and determine whether they are removable, essential, or poles. For poles, determine the order and find the principal part at that pole. [This is Gamelin VI.2 #1a,b]

$$\text{(a)}\quad \frac{z}{(z^2-1)^2} \qquad\qquad \text{(b)}\quad \frac{ze^z}{z^2-1}$$

#9 Beware of Cancellation Find the radius of convergence of the power series for the following functions, expanded about the indicated point. [This is Gamelin VI.2 #2a,b]

$$\text{(a)}\quad \frac{z-1}{z^4-1} \text{ about } z = 3+i \qquad\qquad \text{(b)}\quad \frac{\cos(z)}{z^2 - \pi^2/4} \text{ about } z = 0$$