Math 4010-101 # Homework #2 **Due: Mon., Jan. 31$^{\text{st}}$, 2022**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1. Linear Correspondence practice. Find A and B where...

$$A = \begin{bmatrix} 1 & ? & 1 & ? & ? & 3 & ? & ? \\ 0 & ? & 2 & ? & ? & 1 & ? & ? \\ 2 & ? & 2 & ? & ? & -1 & ? & ? \\ -1 & ? & 1 & ? & ? & 2 & ? & ? \end{bmatrix} \underset{\sim}{\text{RREF}} \begin{bmatrix} 1 & 3 & 0 & -1 & -1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 2 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} ? & ? & -2 & ? & -2 & ? \\ ? & ? & -4 & ? & 5 & ? \\ ? & ? & -6 & ? & 7 & ? \\ ? & ? & -2 & ? & 0 & ? \end{bmatrix} \underset{\sim}{\text{RREF}} \begin{bmatrix} 0 & 1 & -2 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

#2. Define $T : \mathbb{R}^{2\times 2} \to \mathbb{R}[x]$ as follows: $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+2b+3d)x^2 + (-a-2b+c+d)x + (a+2b+c+7d)$

Note that T is a linear transformation (you don't need to prove this).

(a) Write down the standard coordinate matrix for T and find its RREF.
Note that we use $\alpha = \{E_{11}, E_{12}, E_{21}, E_{22}\}$ as our input basis and $\beta = \{1, x, x^2, \dots\}$ as our output basis. Also, you will need to truncate the infinitely many rows of zeros when doing Gaussian elimination.

(b) Find a basis for the kernel and range of T.

(c) What is the nullity and rank of T? Is T 1-1, onto, both, neither?

#3. Consider the following subspaces of $\mathbb{R}^{2\times 3}$:

$$W = \left\{ \begin{bmatrix} -3a-4b+c & b-c & -4a-6b+2c \\ a+b & 2b-2c & a+b \end{bmatrix} \;\middle|\; a,b,c \in \mathbb{R} \right\}$$

$$V = \left\{ \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \;\middle|\; \begin{array}{rcl} a+3b+2c+4d+e+7f & = & 0 \\ 2a+6b+c+5d-e+5f & = & 0 \\ 3a+9b+c+7d-2e+6f & = & 0 \end{array} \right\}$$

(a) Show W and V are subspaces by re-expressing each as either a span or kernel. Pick the most convenient description.

(b) Show $W \subseteq V$.

(c) Find a basis for V.

(d) Find a basis for W. Then extend this to a basis for all of V.

#4. Let V be a vector space over some field $\mathbb{F}$ and let W be a subspace of V.

(a) Show that $\dim(W) \le \dim(V)$. What can be said if $\dim(W) = \dim(V)$?

(b) Show there exists some subspace U such that $W \oplus U = V$.

(c) Let $W = \mathbb{R} \times \{0\} = \{(x,0) \mid x \in \mathbb{R}\}$. Describe all possible subspaces U such that $W \oplus U = \mathbb{R}^2$.