Math 4010-101 **Homework #6** **Due: Wed., Mar. 30^{th}, 2022**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1. First Order Problems: Solve the following initial value problems.

(a) $y' + (\cos(x))y = \cos(x)$, $y(\pi) = 0$

(b) $(x^2 + 1)y' - 2xy = x^2 + 1$, $y(1) = \pi$

#2. Lowering Orders: Convert the following systems of differential equations to an equivalent first order system (using $x_1, x_2, \ldots$ as dependent variable names). State whether the system is linear or not. If the system is linear, write the equivalent first order system in vector-matrix form: $\mathbf{x}'(t) = A(t)\mathbf{x} + \mathbf{g}(t)$.

(a) $(y'')^3 + \sin(y'z') = e^{y+z''}$ and $z''' + e^{2t}y' + \sin(t)z' = 5$

(b) $y''' + t^5y'' + e^{-t^2}y' + \sin(t)y = t^3 - 7$

#3. Just Factoring: Find the general solution of the following homogeneous linear differential equations:

(a) $y^{(5)} - 4y''' = 0$

(b) $y^{(6)} + 6y^{(5)} + 16y^{(4)} + 32y''' + 48y'' + 32y' = 0$

(c) $y^{(4)} + 4y''' + 24y'' + 40y' + 100y = 0$

#4. Making My Own Way: Find a linear homogeneous differential equation with real constant coefficients, whose order is as low as possible, that has the given function as a solution:

(a) $x - e^{3x}$

(b) $e^x \sin(2x)$

(c) $\cos(2x) + 3e^{-x}$