Math 4010-101 # Homework #7 **Due: Wed., Apr. 6^{th}, 2022**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

Assumption: For Cauchy-Euler equations, we assume that the independent variable is positive (i.e., $x > 0$ or $t > 0$).

#1. More than Just Factoring: Find the general solution of the following homogeneous linear differential equations:

(a) $x^3y''' + x^2y'' - 6xy' + 6y = 0$

(b) $x^4y^{(4)} + 10x^3y''' + 32x^2y'' + 54xy' + 36y = 0$

(c) $x^3y''' + 8x^2y'' + 23xy' + 13y = 0$

Note: I suggest handing off the algebra of multiplying out polynomials like $S(S-1)(S-2)$ etc. and factoring to software like Maple or a website like Wolfram Alpha.

#2. Making My Own Way Again: Find a linear homogeneous differential equation of Cauchy-Euler type (with real coefficients), whose order is as low as possible, that has the given function as a solution.

(a) $x^{-3}\ln(x) + x^5$

(b) $x^{-1}\cos(3\ln(x))$

Suggestion: Find the factored polynomial in $S = t\frac{d}{dt}$ that annihilates these functions. Then subtract out $S(S-1)(S-2)$ (i.e., $x^3\frac{d^3}{dx^3}$) followed by subtracting out a multiple of $S(S-1)$ (i.e., $x^2\frac{d^2}{dx^2}$) etc. to find the Cauchy-Euler equation in the form $x^ny^{(n)} + a_{n-1}x^{n-1}y^{(n-1)} + \cdots + a_1xy' + a_0y = 0$.

#3. Fun with Differential Operators: For convenience, let a, b, c be constants and $D = \frac{d}{dx}$.

(a) The differential operators x^2D and Dx^2 are not equal. Find an (operator) equation relating them.

(b) The equation $(x^2D - a)[y] = 0$ is the same as $x^2y' - ay = 0$. Find the general solution of this equation. Separation of variables should work.

(c) Find the second order linear equation equivalent to $(x^2D - a)(x^2D - b)[y] = 0$ (sort of "multiply it out" but be careful about $x^2D \neq Dx^2$): $??y''+??y'+??y = 0$. Find the general solution of such an equation. Consider 3 cases: (1) Distinct real roots, (2) A repeated real root, and (3) A pair of complex conjugate roots.

(d) Find the third order linear equation equivalent to $(x^2D - a)^3[y] = 0$. Also, find the general solution of this equation.

#4. Non-Homogeneous Problems: You might want to use software to help differentiate, determine undetermined coefficients (i.e., solve equations), and factor characteristic polynomials.

(a) Consider $y''' - 3y' - 2y = te^{-t}\sin(3t) - 4e^{-t} + 5t^2 - 2$. Write down the most efficient "guess" for a particular solution as suggested by the method of undetermined coefficients.

Don't worry about determining the coefficients and actually solving. Just write down the "form" for the particular solution.

(b) Solve $t^2y'' + ty' + 4y = \sin(\ln(t^2)) + 3\sqrt{t}$ using the method of undetermined coefficients.

(c) Solve $y'' - 4y' + 3y = t^3 + 3$ using

(1) the method of undetermined coefficients

(2) variation of parameters.