

Munkres' Topology §2 Problem #2 (pages 20-21): Let  $f : A \rightarrow B$  and let  $A_i \subset A$  and  $B_i \subset B$  for  $i = 0$  and  $i = 1$ . Show that  $f^{-1}$  preserves inclusions, unions, intersections, and differences of sets:

(a)  $B_0 \subset B \implies f^{-1}(B_0) \subset f^{-1}(B).$

(b)  $f^{-1}(B_0 \cup B_1) = f^{-1}(B_0) \cup f^{-1}(B_1).$

(c)  $f^{-1}(B_0 \cap B_1) = f^{-1}(B_0) \cap f^{-1}(B_1).$

(d)  $f^{-1}(B_0 - B_1) = f^{-1}(B_0) - f^{-1}(B_1).$

Show that  $f$  preserves inclusions and unions only:

(e)  $A_0 \subset A_1 \implies f(A_0) \subset f(A_1).$

(f)  $f(A_0 \cup A_1) = f(A_0) \cup f(A_1).$

(g)  $f(A_0 \cap A_1) \subset f(A_0) \cap f(A_1)$ ; show equality holds if  $f$  is injective.

(h)  $f(A_0 - A_1) \supset f(A_0) - f(A_1)$ ; show equality holds if  $f$  is injective.