Math 4710/5710-101 # Homework #3 **Due: Wed., Sept. 10^{th}, 2014**

CARDNALITY: If there is an injection from X into $\mathbb{N} = \mathbb{Z}_{\geq 0}$ (the natural numbers), we say that X is **countable**. If there is a bijection between X and $\mathbb{N}$, then X is countably *infinite* otherwise it is *finite*.

Munkres' §7 problem #5 (page 51). Determine which sets are countable. Justify your answers!

(a) The set A of all functions $f : \{0, 1\} \to \mathbb{Z}_{>0}$.

(b) [Let n be some fixed positive integer.] The set B_n of all functions $f : \{1, \ldots, n\} \to \mathbb{Z}_{>0}$.

(c) The set $C = \bigcup_{n \in \mathbb{Z}_{>0}} B_n$.

(d) The set D of all functions $f : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$.

(e) The set E of all functions $f : \mathbb{Z}_{>0} \to \{0, 1\}$.

(f) The set F of all functions $f : \mathbb{Z}_{>0} \to \{0, 1\}$ that are **eventually zero**. This means that there is some $N > 0$ such that $f(x) = 0$ for all $x > N$.

(g) The set G of all functions $f : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ that are eventually 1.

(h) The set H of all functions $f : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ that are eventually constant.

(i) The set I of all two-element subsets of $\mathbb{Z}_{>0}$.

(j) The set J of all finite subsets of $\mathbb{Z}_{>0}$.

Grad students: Do all of these. Undergrads: You may skip 2 of your choice.