Math 4710/5710-101 | # Homework #7 | **Due: Wed., Oct. 22^{nd}, 2014**

Recall that $\mathbb{R}^\omega = \mathbb{R} \times \mathbb{R} \times \cdots$ is the set of sequences of real numbers: $\mathbf{x} = (x_1, x_2, x_3, \dots)$.

Our default topology on $\mathbb{R}^\omega$ is the product topology. However, it can also be given the box topology or the uniform topology.

Recall that the *uniform topology* is the topology induced from the uniform metric. The *uniform metric* on $\mathbb{R}^\omega$ is the function $\bar{\rho}(\mathbf{x}, \mathbf{y}) = \sup\{\bar{d}(x_i, y_i) \mid i = 1, 2, \dots\}$ where $\mathbf{x} = (x_1, x_2, \dots)$, $\mathbf{y} = (y_1, y_2, \dots)$, and $\bar{d}(a, b) = \min\{|a - b|, 1\}$.

Let $\mathbb{R}^\infty = \{\mathbf{x} \in \mathbb{R}^\omega \mid \mathbf{x} \text{ is eventually zero }\}$. So $\mathbf{x} = (x_1, x_2, \dots) \in \mathbb{R}^\infty$ implies that there exists some $N \in \mathbb{Z}_{>0}$ such that $x_n = 0$ for all $n \geq N$. So in other words, the coordinates of $\mathbf{x}$ are non-zero only finitely many times.

THE PROBLEM: Determine the closure of $\mathbb{R}^\infty$ in $\mathbb{R}^\omega$ in the product, box, and uniform topologies.