Math 4710/5710 # Homework #1 **Due: Wed., Jan. 22$^{\text{nd}}$, 2025**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

Grads. – please do both versions – regarding problems #1b, #1c, #2b, and #2c.

#1 Preimages Let $f : X \to Y$. Preimages preserve inclusions, unions, intersections, and differences of sets.

For regular versions: Let $B_i \subseteq Y$ where $i = 0, 1$.

For grad. versions: Let $B_i \subseteq Y$ where $i \in I$ and I is some arbitrary index set.

Note: Recall that $x \in f^{-1}(B_i)$ if and only if $f(x) \in B_i$.

(a) Show $B_0 \subseteq B_1$ implies $f^{-1}(B_0) \subseteq f^{-1}(B_1)$.

(b) Show $f^{-1}(B_0 \cup B_1) = f^{-1}(B_0) \cup f^{-1}(B_1)$.
[Grads.] Also show $f^{-1}\left(\bigcup_{i\in I} B_i\right) = \bigcup_{i\in I} f^{-1}(B_i)$.

(c) Show $f^{-1}(B_0 \cap B_1) = f^{-1}(B_0) \cap f^{-1}(B_1)$.
[Grads.] Also show $f^{-1}\left(\bigcap_{i\in I} B_i\right) = \bigcap_{i\in I} f^{-1}(B_i)$.

(d) Show $f^{-1}(B_0 - B_1) = f^{-1}(B_0) - f^{-1}(B_1)$.

#2 Images Let $f : X \to Y$. Images preserve inclusions and unions of sets.

For regular versions: Let $A_i \subseteq X$ where $i = 0, 1$.

For grad. versions: Let $A_i \subseteq X$ where $i \in I$ and I is some arbitrary index set.

Note: Recall that $x \in f(A_i)$ if and only if there exists some $a \in A_i$ such that $f(a) = x$.

(a) Show $A_0 \subseteq A_1$ implies $f(A_0) \subseteq f(A_1)$.

(b) Show $f(A_0 \cup A_1) = f(A_0) \cup f(A_1)$.
[Grads.] Also show $f\left(\bigcup_{i\in I} A_i\right) = \bigcup_{i\in I} f(A_i)$.

(c) $f(A_0 \cap A_1) \subseteq f(A_0) \cap f(A_1)$; show equality holds if f is injective.
[Grads.] Also show $f\left(\bigcap_{i\in I} A_i\right) \subseteq \bigcap_{i\in I} f(A_i)$; show equality holds if f is injective.

(d) $f(A_0 - A_1) \supseteq f(A_0) - f(A_1)$; show equality holds if f is injective.