Math 4710/5710 **Homework #5** **Due: Mon., Feb. 24th, 2025**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 A Problem of Intervals: Let $Y = [-1, 1]$ be thought of as a subspace of $\mathbb{R}$ (with the standard topology). For each of the following sets: If open, explain why. If not open, explain why not.

$A = \{x \in \mathbb{R} \mid \frac{1}{2} < |x| < 1\}$, $B = \{x \in \mathbb{R} \mid \frac{1}{2} < |x| \leq 1\}$, $C = \{x \in \mathbb{R} \mid \frac{1}{2} \leq |x| < 1\}$, $D = \{x \in \mathbb{R} \mid \frac{1}{2} \leq |x| \leq 1\}$, and $E = \{x \in \mathbb{R} \mid 0 < |x| < 1 \text{ and } 1/x \notin \mathbb{Z}_{>0}\}$.

[Grad.] Answer the same questions for Y thought of as a subspace of (a) $\mathbb{R}_\ell$ (the lower limit topology) and of (b) $\mathbb{R}_K$ (the K-topology).

#2 Open Your Map: Let X and Y be topological spaces and $f : X \to Y$. We say f is an *open map* if for every open set $U \subseteq X$, we have $f(U)$ is open (in Y).

Next, fix some index i (between 1 and n) and recall that $\pi_i : X_1 \times X_2 \times \cdot \times X_n \to X_i$ is the projection map $\pi_i(x_1, \dots, x_n) = x_i$ onto the i^{th} coordinate. Show π_i is an open map.

#3 Lines are Easy - Right? Consider $Y = \{x \times y \in \mathbb{R}^2 \mid y = 1 - x\}$ (a line with negative slope).[1] If we think of Y as a subspace of $\mathbb{R}^2$ with the standard topology, Y "look like" the reals with the standard topology. [Technically, by "looks like" I mean "homeomorphic" – more on this later.]

Describe the topology on Y if we think of it as a subspace of $\mathbb{R}_\ell \times \mathbb{R}$ (lower limit product standard topology). Then describe the topology on Y if we think of it as a subspace of $\mathbb{R}_\ell \times \mathbb{R}_\ell$ (lower limit product lower limit topology).

#4 Getting Things In Order: [Grad.] Let $\mathbb{R}^2_{\text{lex}}$ denote $\mathbb{R} \times \mathbb{R}$ with the order topology coming from the lexicographic ordering of the plane. Let $\mathbb{R}_d$ denote the real numbers with the discrete topology and give $\mathbb{R}_d \times \mathbb{R}$ the product topology (discrete by standard). Show $\mathbb{R}^2_{\text{lex}}$ and $\mathbb{R}_d \times \mathbb{R}$ have the same topology.

Note: In class we sketched out that $\mathbb{R}^2_{\text{lex}}$'s topology is strictly finer than the standard topology on $\mathbb{R}^2$. This problem better explains why this is the case.

[1] I am using the weird notation: $x \times y$ instead of the usual (x, y) since this problem involves intervals. Sorry!