Math 4710/5710 # Homework #6 **Due: Wed., Mar. 5th, 2025**

Please remember when submitting any work via email or in person to...

PUT YOUR NAME ON YOUR WORK!

#1 Getting Closure: Let A, B, and C_i (where $i \in I$) be subsets of a topological space X. Show...

(a) If $A \subseteq B$, then $\overline{A} \subseteq \overline{B}$

(b) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

(c) $\displaystyle\bigcup_{i \in I} \overline{C_i} \subseteq \overline{\bigcup_{i \in I} C_i}$

(d) Give an example of the containment in part (c) being proper (i.e., not equal).

#2 Open to Inequality (Shame on you Hausdorff!): Let X be a Hausdorff (i.e., T_2) space.
Show the diagonal $\Delta = \{(x, x) \mid x \in X\}$ is closed in $X \times X$ (with the product topology).

#3 Weird Closure: [Grad.] Recall $\mathbb{R}_\ell$ is the real numbers equipped with the lower limit topology. Let $\mathbb{R}_{\text{rat.}-\ell}$ be the real numbers equipped with the topology generated by the basis $\mathcal{C} = \{[a, b) \mid a, b \in \mathbb{Q}\}$. We could call this the rational lower limit topology. *Note:* We previously showed that the topology on $\mathbb{R}_\ell$ is strictly finer than that of $\mathbb{R}_{\text{rat.}-\ell}$.

Determine the closures of $A = (0, \sqrt{2})$ and $B = (\sqrt{2}, 4)$ in the (a) standard topology, (b) lower limit topology, and (c) rational lower limit topology.

#4 Continuous Products: Let $f : A \to B$ and $g : C \to D$ be continuous functions between topological spaces. Consider $f \times g : A \times C \to B \times D$ defined by $(f \times g)(a, c) = (f(a), g(c))$. Prove $f \times g$ is continuous.