Math 5160 **Homework #1** **Due: Fri., Jan. 24^{th}, 2020**

#1 Basics Initial Calculations

(a) Let $z = 3 - 4i$ and $w = -1 + 2i$. Compute $z^2, \bar{z}, |z|, 1/z, wz, w + z, z/w$.

(b) Solve $z^4 - 5z^2 + 6 = 0$. *Note:* Find <u>all</u> complex solutions.

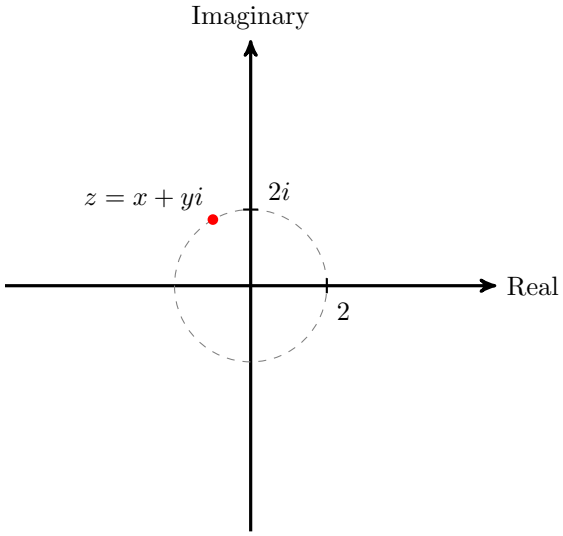
(c) Assume $z_1 = x_1 + y_1 i$ has positive real and imaginary parts and let $z_2 = \overline{z_1}$. What are $z_1 + z_2$ and $z_1 - z_2$? Plot $z_1, z_2, z_1 + z_2, z_1 - z_2$ together.

(d) Plot the region $|z - 2i| \leq 2$.

(e) Write i, $1 - \sqrt{3}i$, and $\sqrt{3} + i$ in polar form, multiply them together (in polar), then convert back from polar to standard form.

(f) Find the 3^{rd} roots of unity (in polar and standard form). Then solve $z^3 = -1 + i$.

#2 Plotting Against You Let $z = x + yi$ be as shown below:



(a) Plot and label $-z$

(b) Plot and label $\bar{z}$

(c) Plot and label $1/z$

(d) Plot and label z^2

#3 Transformers Go! In class, we showed that $w = (z - (1 + 2i))e^{i\pi/4} + (1 + 2i)$ rotates the complex plane 45° about the point $(1, 2) = 1 + 2i$. Also, $w = -\bar{z} + 2$ reflects the complex plane across the line $\text{Re}(z) = 1$. Give similar formulas for the following transformations:

(a) Rotate the plane 30° about the point $-1 + i$.

(b) Reflect across the line through $1 + \sqrt{3}i$ and $3 - \sqrt{3}i$.