Math 5160 # Homework #5 **Due: Wed., Mar. 18^{th}, 2020**

#1 Let's Get Integrating: First, parameterize the given curve. Then use that parameterization to compute the integral.

(a) Let C be the line segment from $A = 1 - i$ to $B = -2i$. $\displaystyle\int_C (3z^2 - 4z - 5)\,dz$

(b) Let C be the upper-half of the circle $|z+1| = 2$, oriented counter-clockwise. $\displaystyle\int_C \frac{z^2}{1+z}\,dz$

#2 Let's Estimate Use a ML-estimate to show that $\displaystyle\left|\oint_{|z-1|=1} \frac{e^z}{z+1}\,dz\right| \leq 2\pi e^2$.

#3 Fundamentally Flawed: Recall the fundamental theorem of calculus (for analytic functions) in Gamelin IV.2.

(a) Recompute #1(a) using the FTC for analytic functions.

(b) Let C be parameterized by $\gamma(t) = (\sin(2t) + t^2) + i(3\cos(t) + 1)$ where $-\pi \leq t \leq 0$.
Compute $\displaystyle\int_C (2z + \cos(z) + 6e^{3z})\,dz$.

#4 Watch Out! Work with Cauchy: Compute the following integrals using the Cauchy integral formula and its consequences.

(a) Compute $\displaystyle\oint_{|z|=2} \frac{2z-1}{z^2(z+3)}\,dz$ and $\displaystyle\oint_{|z-3|=2} \frac{2z-1}{z^2(z+3)}\,dz$.

(b) Compute $\displaystyle\oint_C \frac{dz}{z^2+4}$ where C is any counter-clockwise oriented closed loop which avoids $z = \pm 2i$. There are 4 distinct cases (consider when each of the points $z = \pm 2i$ are inside or outside your curve). Draw a representative curve in these 4 distinct situations and compute the value of this integral in each case.

(c) Compute $\displaystyle\int_0^{2\pi} \frac{1}{3 + \sin(\theta) + \cos(\theta)}\,d\theta$.
Note: This problem is Fisher 2.3 #6. Convert the above integral back to a contour integral. Then apply the Cauchy integral formula. You might want to use technology to help with the algebraic manipulations and factoring. Examples in Fisher 2.3 should help guide your solution.

#5 Seems Pretty Fundamental: Prove that a complex polynomial with no zeros must be constant. To do this use Cauchy's Theorem and an ML-estimate. Begin by supposing $P(z)$ is a non-constant polynomial then write $P(z) = P(0) + zQ(z)$. Divide this by $zP(z)$ to get $\displaystyle\frac{1}{z} = \frac{P(0)}{zP(z)} + \frac{Q(z)}{P(z)}$. Integrate around a circle of radius R and see what happens as $R \to \infty$. Contradiction?

You may use the following:

Lemma: Suppose $P(z)$ is a non-constant complex polynomial.
There exists positive real numbers c and R_0 such that $|P(z)| \geq c|z|$ for all $|z| \geq R_0$.