Problems #1–#6 are Homework #7 and are due Wednesday, April 8.

Problems #7-#9 are Homework #8 and are due Wednesday, April 15.

#1 Bumping Up Against Infinity Find the radius of convergence of the power series for the following functions if they were expanded about the given point. [This is Gamelin V.4 #1a,b,c]

(a)
$$\frac{1}{z-1}$$
 about $z = i$ (b) $\sec(z) = \frac{1}{\cos(z)}$ about $z = 0$ (c) $\operatorname{sech}(z) = \frac{1}{\cosh(z)}$ about $z = 0$

#2 Standard Series Manipulation Find the power series expansion of the principal branch of $\arctan(z)$ expanded about z = 0. What is the radius of convergence of this series? *Hint:* Find an expansion for the derivative of arctan using a geometric series and then integrate it. [This is Gamelin V.4 #7]

#3 Infinite Power Find the power series expansion about $z = \infty$ for $\frac{z^2}{z^3 - 1}$. Where does this series converge? [This is part of Gamelin V.5 #1]

#4 Who Needs Polynomial Long Division? Calculate up to (and including) the fifth order terms in the power series expansion about z = 0 (where do these expansions converge?): [This is parts of Gamelin V.6 #2 and #3]

(a)
$$\frac{z}{\sin(z)}$$
 (b) $\frac{e^z}{1+z}$

#5 Establishing Order For each of the following functions, first, find the zeros and their orders. Next, determine if they are analytic at ∞ and determine orders of any zeros there. [This is Gamelin V.7 #1d,f and #2d,f]

(a)
$$\cos(z) - 1$$
 (b) $\frac{\cos(z) - 1}{z}$

#6 This is Easy – Don't Make it Hard Show that all of the zeros of sin(z) and tan(z) are simple. [This is Gamelin V.7 #3]

#7 Laurent Has Decomposed Consider $f(z) = \frac{1}{z^2 - z}$ [This is Gamelin VI.1 #1a and #2a]

- (a) Find all of the Laurent expansions centered at z = 0.
- (b) Find the Laurent expansion centered at z = -1 that converges at z = 1/2. Determine the largest open set where this expansion converges.
- **#8 I am Just a Simple Pole in a Complex Plane** Find the isolated singularities of the following functions and determine whether they are removable, essential, or poles. For poles, determine the order and find the principal part at that pole. [This is Gamelin VI.2 #1a,b]

(a)
$$\frac{z}{(z^2-1)^2}$$
 (b) $\frac{ze^z}{z^2-1}$

#9 Beware of Cancellation Find the radius of convergence of the power series for the following functions, expanded about the indicated point. [This is Gamelin VI.2 #2a,b]

(a)
$$\frac{z-1}{z^4-1}$$
 about $z = 3+i$ (b) $\frac{\cos(z)}{z^2-\pi^2/4}$ about $z = 0$