Math 5160 | **Homework #2** | **Do after Gamelin I.8**

#1 Basics More "Basic" Calculations

(a) Compute e^z for $z = -\frac{i\pi}{3}$, $\frac{1}{2} - \frac{i\pi}{4}$, and $-1 + \frac{i3\pi}{2}$.

(b) Compute $\mathrm{Log}(4+4i)$ and $\mathrm{Log}(-10i)$. Then also compute $\log(4+4i)$ and $\log(-10i)$.

(c) Compute $(-1)^i$ and indicate what the principal branch yields.

(d) Compute $\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)^{1+i}$ and indicate what the principal branch yields.

(e) Compute $\sin(2i)$ and $\tan(2i)$.

(f) Compute $\cosh(i\pi/3)$ and $\sinh(i\pi/3)$.

#2 Exponentially Difficult Graphs Sketch each of the following figures and its image under the exponential map $w = e^z$. Indicate images of vertical and horizontal lines in your sketch.

(a) The vertical strip $0 < \mathrm{Re}(z) < 1$.

(b) The horizontal strip $5\pi/3 < \mathrm{Im}(z) < 8\pi/3$.

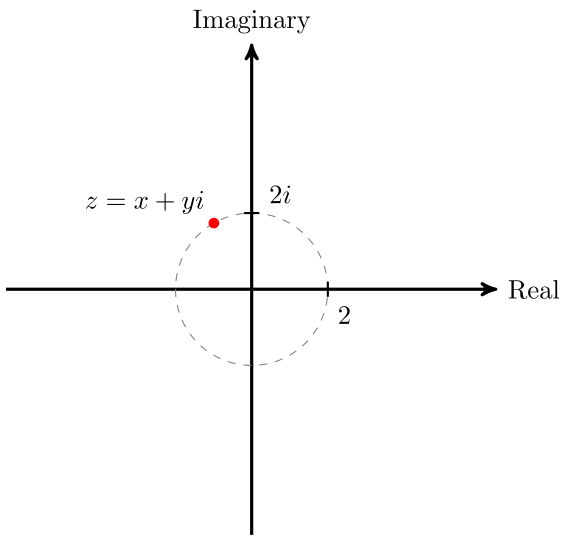
(c) The rectangle $0 < x < 1$, $0 < y < \pi/4$.

(d) [**Extra Credit**] The disk $|z| \leq \pi/2$.

(e) [**Extra Credit**] The disk $|z| \leq \pi$.

(f) [**Extra Credit**] The disk $|z| \leq 3\pi/2$.

#3 Log Your Complaint Let $z = x + yi$ be as shown below:



(a) Is $\mathrm{Log}(z^2) = 2\mathrm{Log}(z)$? Explain why or why not.

(b) In general, for which z do we have $\mathrm{Log}(z^2) = 2\mathrm{Log}(z)$? Justify your answer.

(c) For $z \neq 0$, it is always that case that $e^{\log(z)} = z$ (and $e^{\mathrm{Log}(z)} = z$). But obviously, "$\log(e^z) = z$" cannot hold because log is multivalued. When is it the case that $\mathrm{Log}(e^z) = z$?

#4 A Tangential Issue One can show that $\arctan(z) = \frac{i}{2}\log\left(\frac{1-iz}{1+iz}\right)$ when $z \neq \pm i$. The principal branch of inverse tangent is $\mathrm{Arctan}(z) = \frac{i}{2}\mathrm{Log}\left(\frac{1-iz}{1+iz}\right)$ when $z \neq \pm i$.

(a) Derive the above formula for $\arctan(z)$.

(b) Show that it is never the case that $\tan(z) = \pm i$. [Taking into account our formula for $\arctan(z)$, which is defined for all $z \neq \pm i$, and the fact that $\tan(\arctan(z)) = z$, we get that the range of $\tan(z)$ is $\mathbb{C} - \{\pm i\}$.]