Math 5160 # Homework #7 **Do after Gamelin V.7**

#1 Bumping Up Against Infinity Find the **radius of convergence** of the power series for the following functions if they were expanded about the given point. [No need to actually expand.]

(a) $\dfrac{1}{z-1}$ about $z = i$ (b) $\sec(z) = \dfrac{1}{\cos(z)}$ about $z = 0$ (c) $\operatorname{sech}(z) = \dfrac{1}{\cosh(z)}$ about $z = 0$

#2 Standard Series Manipulation Find the power series expansion of the principal branch of $\arctan(z)$ expanded about $z = 0$. What is the radius of convergence of this series?

Hint: Find an expansion for the derivative of arctan using a geometric series and then integrate it.

Note: This is more-or-less a Calculus II problem!

#3 Infinite Power Find the power series expansion about $z = \infty$ for $\dfrac{z^2}{z^3-1}$. Where does this series converge?

#4 Who Needs Polynomial Long Division? For each of the following functions, calculate up to (and including) the fifth order terms in their power series expansions about $z = 0$. Also, where do these series converge?

(a) $\dfrac{z}{\sin(z)}$ (b) $\dfrac{e^z}{1+z}$

#5 Establishing Order For each of the following functions, first, find the zeros and their orders. Next, determine if these functions are analytic at ∞ and determine orders of any zeros there.

(a) $\cos(z) - 1$ (b) $\dfrac{\cos(z)-1}{z}$

#6 This is Easy – Don't Make it Hard Show that all of the zeros of $\sin(z)$ and $\tan(z)$ are simple.