Math 5230 # Homework #1 **Due: Fri., Aug. 26^{th}, 2016**

#1 RREF First, (1) use Gaussian elimination (as defined on the handout) to compute the RREF of A. Then, (2) give a vector version of the general solution of $A\mathbf{x} = \mathbf{0}$. Finally, (3) suppose $A = [C \;:\; \mathbf{b}]$ (C is all but the last column and $\mathbf{b}$ is the last column of A). Give a vector form of the general solution of $C\mathbf{x} = \mathbf{b}$.

$$\text{(a)}\quad A = \begin{bmatrix} 1 & -2 & 0 & 2 & 3 \\ 2 & -4 & 1 & 5 & 4 \\ -1 & 2 & 1 & -1 & -5 \end{bmatrix} \qquad\qquad \text{(b)}\quad A = \begin{bmatrix} 0 & 1 & 3 & 1 \\ 2 & 0 & 4 & 2 \\ 1 & -1 & -1 & 3 \end{bmatrix}$$

#2 PLU and Partial Pivoting First, (1) use partial pivoting to row reduce A (to REF not RREF), and compute a PLU decomposition for A. Then (2) use this decomposition to solve $A\mathbf{x} = \mathbf{b}$.

$$\text{(a)}\quad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \qquad\qquad \text{(b)}\quad A = \begin{bmatrix} 1 & 4 & 1 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 1 \\ 4 & 1 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 3 \end{bmatrix}$$

#3 Matrix Mechanics First, (1) give a formula for a matrix which performs the given list of row operations. For example: Swapping rows 1 and 2 is $E = I - E_{11} - E_{22} + E_{12} + E_{21}$. Then, (2) give a formula for its inverse (for my example $E^{-1} = E = I - E_{11} - E_{22} + E_{12} + E_{21}$). Next, (3) what happens if you multiply a matrix by your E on the right? (my example swaps columns 1 and 2). Finally, (4) give a concrete example to show off this behavior (for my example I could take some 2×2 matrix and show how E swaps its rows and columns when multiplying on the left or right).

(a) Find a matrix E which swaps rows 3 and 4 and then scales row 2 by 5.

(b) Find a matrix E which swaps rows 1 and 4 and then adds -2 times row 2 to row 3.

#4 Matrix Mechanics II: Matrices Arise Let $B = I - E_{11} + 3E_{22} - E_{33} - E_{99} + E_{39} + E_{93}$. What does BA do to A? What does AB do to A? (Assume that A and B are appropriately sized.[1])

#5 Matrix Mechanics III: Revenge of the Matrices Let $\mathbb{R}^{n\times n}$ be the collection of all square $n \times n$ matrices with real entries. We say that $A \in \mathbb{R}^{n\times n}$ is in the **center** of $\mathbb{R}^{n\times n}$ if $AB = BA$ for all $B \in \mathbb{R}^{n\times n}$. In other words, central matrices commute with all matrices of the same size.

Show that the A is in the center if and only if $A = cI$ for some $c \in \mathbb{R}$ (i.e. the central matrices are exactly the scalar multiples of the identity). Please do this using E_{ij}'s.

[1] In this day and age is it ok to talk about "appropriately sized" matrices? Shouldn't we accept a matrix just the way it is no matter how big or small it is? I think Dr. Cook might be encouraging bias. I should probably fill out a form.