Math 5230 — # Homework #4 — **Due: Wed., Sept. 21^{st}, 2022**

#1 Diagonalize For each of the following matrices, find the characteristic polynomial. Then find the eigenvalues and a basis for each eigenspace. State the algebraic and geometric multiplicities of each eigenvalue. Note the determinant and trace of the matrix. Finally, comment whether the matrix is diagonalizable or not (over $\mathbb{C}$ – the complex numbers). If it's diagonalizable over $\mathbb{C}$, state the which of the following fields allow us to diagonalize: $\mathbb{Q}$ – the rationals, $\mathbb{R}$ – the reals, $\mathbb{C}$ – the complexes.

(a) $A_1 = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

(b) $A_2 = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$

(c) $A_3 = \begin{bmatrix} 0 & -1 \\ 9 & 6 \end{bmatrix}$

(d) $A_4 = \begin{bmatrix} 4 & 0 & 2 \\ 1 & 3 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

(e) $A_5 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$

(f) $A_6 = \begin{bmatrix} 0 & 1 & -1 \\ -1 & -1 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

#2 Eigenstuff Let A be an $n \times n$ matrix and assume the characteristic polynomial completely factors (for example, work over the complex numbers).

(a) Explain why 0 is an eigenvalue if and only if A is a singular matrix (i.e. A^{-1} does not exist).

(b) Explain why the determinant of A is the product (counting multiplicities) of the eigenvalues.
[For example: If the characteristic polynomial of A is $f(t) = (t-5)^2(t-3)$ then $\det(A) = 5^2 \cdot 3 = 75$.]
What can be said about the trace of A?

(c) Suppose the characteristic polynomial of A is $f(t) = (t-4)^n$. What is the trace and determinant of A? If A is diagonalizable, what is A?

#3 Gram-Schmidt Use the Gram-Schmidt algorithm to produce an orthogonal basis for the column spaces of the following matrices. [Don't normalize the vectors.]

(a) $B_1 = \begin{bmatrix} 1 & 2 & 0 & -2 & 1 \\ 1 & 2 & 2 & 4 & -1 \\ -1 & -2 & -1 & -1 & 1 \end{bmatrix}$

(b) $B_2 = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$