Math 5230 # Homework #2 **Due: Wed., Sept. 4^{th}, 2024**

#1 Row, Column, Null Let $A = \begin{bmatrix} 1 & 2 & 0 & 3 & 2 & -1 \\ 2 & 4 & 3 & 0 & 0 & 2 \\ -1 & -2 & 1 & -5 & 1 & -2 \\ 3 & 6 & -2 & 13 & 0 & 3 \end{bmatrix}$

Find a basis for (i) $\text{Row}(A)$, (ii) $\text{Col}(A)$, and (iii) $\text{Null}(A)$.

#2 Linear Correspondence Fill in the missing entries. A is a matrix with real entries and R is its RREF.

(a) $A = \begin{bmatrix} 0 & ? & 1 & ? & ? & 3 & ? & ? \\ 1 & ? & -1 & ? & ? & 2 & ? & ? \\ 2 & ? & 1 & ? & ? & 1 & ? & ? \\ 1 & ? & -1 & ? & ? & 2 & ? & ? \end{bmatrix} \overset{\text{RREF}}{\sim} R = \begin{bmatrix} 1 & -1 & 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 3 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} ? & ? & -2 & ? & 4 & ? \\ ? & ? & -2 & ? & 1 & ? \\ ? & ? & -2 & ? & 6 & ? \\ ? & ? & -2 & ? & -1 & ? \end{bmatrix} \overset{\text{RREF}}{\sim} R = \begin{bmatrix} 0 & 1 & -2 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 2 & ? & ? & 4 & ? & 1 \\ 1 & ? & ? & 0 & ? & 2 \\ 1 & ? & ? & 4 & ? & 1 \end{bmatrix} \overset{\text{RREF}}{\sim} R = \begin{bmatrix} ? & 2 & ? & 2 & 1 & ? \\ ? & 0 & ? & 2 & 3 & ? \\ ? & 0 & ? & 0 & 0 & ? \end{bmatrix}$

#3 Coordinates, Spanning, Bases Let $\beta = \{E_{11}, E_{12}, E_{21}, E_{22}\}$ be the standard basis for $\mathbb{R}^{2\times 2}$.

[*Note:* As discussed in class, even though β is a set, treat it like a list and maintain the order as shown above.]

For example, the β-coordinates of the matrix $\begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix}$ are $\left[\begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix}\right]_\beta = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 5 \end{bmatrix}$.

Let $A_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $A_3 = \begin{bmatrix} -1 & -2 \\ -1 & 2 \end{bmatrix}$, $A_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, and $A_5 = \begin{bmatrix} 1 & 3 \\ 6 & 10 \end{bmatrix}$.

(a) Does A_4 belong to $\text{Span}\{A_1, A_2, A_3\}$?

(b) Let $S = \{A_1, A_2, A_3, A_4\}$ and $W = \text{Span}(S)$. Find a subset of S which forms a basis for W.

(c) Using your basis found in part (b), find coordinates for $A_1, \ldots, A_5$.

(d) Extend $\{A_5\}$ to a basis for W.