Math 5230 # Homework #6 **Due: AFTER BREAK**

#1 Linear Independence and Spanning Suppose V has a basis $\beta = \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ (distinct vectors).
Let $S = \{\mathbf{u}+\mathbf{v}, \mathbf{v}+\mathbf{w}, \mathbf{w}+\mathbf{u}\}$ and $T = \{\mathbf{u}+\mathbf{v}, \mathbf{u}-\mathbf{v}, \mathbf{w}\}$.

Additional Assumption: $\mathrm{char}(\mathbb{F}) \neq 2$.

(a) Prove S is linearly independent (using the definition of independence).

(b) Prove T spans V (using the definition of span).

(c) Is $S \cup T$ linearly independent? Does it span V? Explain.

(d) Is $S \cap T$ linearly independent? Does it span V? Explain.

#2 Bases Consider $W = \left\{ \begin{bmatrix} -3a-4b+c & b-c & -4a-6b+2c \\ a+b & 2b-2c & a+b \end{bmatrix} \;\middle|\; a,b,c \in \mathbb{R} \right\}$ and

$$V = \left\{ \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \;\middle|\; \begin{array}{rcl} a+3b+2c+4d+e+7f & = & 0 \\ 2a+6b+c+5d-e+5f & = & 0 \\ 3a+9b+c+7d-2e+6f & = & 0 \end{array} \right\}.$$

(a) Both W and V are subspaces of $\mathbb{R}^{2\times 3}$. Prove that they are subspaces by re-expressing each as either a span or kernel (i.e., null space). Pick the most convenient description.

Note: Re-expressing as a span or kernel would prove we have a subspace since spans and kernels are always subspaces.

(b) Prove that $W \subseteq V$.

(c) Find a basis for V.

(d) Find a basis for W and then extend this to a basis for all of V.