Math 1030 **Final Exam – Section 101** **December 12^{th}, 2008**

PART I. ANSWER KEY

1. (10 points) Given $f(x)$, compute the derivative $f'(x)$.

(a) $f(x) = x^3e^x + \ln(x^4) + 5x - 2$ so that $f(x) = x^3e^x + 4\ln(x) + 5x - 2$

(Use the product rule for the first term.)
$f'(x) = 3x^2e^x + x^3e^x + \dfrac{4}{x} + 5$

(b) $f(x) = e^{-x^2} + \ln(x^2 + 1)$

(Use the chain rule for both terms.)
$f'(x) = e^{-x^2}(-2x) + \dfrac{1}{x^2+1}(2x)$

2. (10 points) Compute the following indefinite integrals.

(a) $\displaystyle\int 1 + \frac{1}{x} + \frac{1}{x^2}\,dx$

$\displaystyle= \int 1 + \frac{1}{x} + x^{-2}\,dx = x + \ln|x| + \frac{x^{-1}}{-1} + C$ (where C is an arbitrary constant.)

(b) $\displaystyle\int e^{-2x}\,dx$

Use the u-substitution: $u = -2x$ so that $du = -2dx$ and thus $-1/2\,du = dx$.
Therefore, $\displaystyle\int e^{-2x}\,dx = \int e^u\left(-\frac{1}{2}\right)du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-2x} + C$ (where C is an arbitrary constant.)

(c) $\displaystyle\int (x^2 - 2x + 5)^2(x-1)\,dx$

Use the u-substitution: $u = x^2 - 2x + 5$ so that $du = (2x-2)dx$ and thus $1/2\,du = (x-1)dx$.
Therefore, $\displaystyle\int (x^2 - 2x + 5)^2(x-1)\,dx = \int u^2\left(\frac{1}{2}\right)du = \frac{1}{2}\cdot\frac{u^3}{3} + C = \frac{(x^2-2x+5)^3}{6} + C$ (where C is an arbitrary constant.)