Math 1030 # Test #2 — ANSWER KEY **October** $13/14^{th}$**, 2008**

Please see the corresponding Excel worksheet for calculation details.

1. (12 points) When Clyde's Discount Cycles sells Ninja motorcycles for $3,000 each, they can sell only 4. However, when they set the price at $1,500, they can sell 36 motorcycles. On the other hand Clyde's supplier will only supply 12 Ninjas at $1,250 a piece but they will supply 48 motorcycles at $2,000 each. Assume that Clyde's supply and demand functions are linear.

(a) Find the equation of the demand function. **Remember to use** p**'s and** q**'s!!!**

Using Excel : We have $(q, p) = (4, 3000)$ and $(q, p) = (36, 1500)$. Entering this data into Excel, graphing, and adding a linear trendline we find that $p = -46.875q + 3187.5$

By Hand : Slope = Rise/Run so $m = (3000-1500)/(4-36) = -1500/32 = -375/8$ Using point-slope, we have $p - 3000 = -(375/8)(q - 4)$ or in slope-intercept form: $p = (375/8)q + 6375/2$ (in decimals... $p = 46.875q + 3187.5$).

(b) Find the equation of the supply function.

Using Excel : We have $(q, p) = (12, 1250)$ and $(q, p) = (48, 2000)$. Entering this data into Excel, graphing, and adding a linear trendline we find that $p = 20.833q + 1000$

By Hand : Slope = Rise/Run so $m = (2000 - 1250)/(48 - 12) = 750/36 = 125/6$ Using point-slope, we have $p - 1250 = (125/6)(q - 12)$ or in slope-intercept form: $p = (125/6)q + 1000$ (in decimals... $p = 20.833q + 1000$).

(c) Find all Market equilibria. In a sentence or two, describe how you found the equilibria or if there are none, explain how you arrived at your conclusion.

The only market equilibrium is $(q, p) = (32.46, 1676.15)$.

EXPLANATION: To find the market equilibrium we compute the difference between supply and demand and goal seek this difference to be zero. [*Since the supply and demand functions are linear, we can have at most 1 equilibrium – so there is no need to graph them and look for a second equilibrium.*]

2. (13 points) Kenny's Overpriced Warehouse has determined that the demand function for their sofas is given by $p = -100q + 6000$. The Warehouse has also concluded that the costs associated with selling these sofas is linear. Their fixed costs being $2,500 and variables costs are $750 per sofa.

(a) Find the revenue function. $R(q) = pq = (-100q + 6000)q = -100q^2 + 6000q$

(b) Find the cost function. $C(q) = 2500 + 750q$

(c) Find the profit function. $P(q) = R(q) - C(q) = (-100q^2 + 6000q) - (2500 + 750q)$ simplifying we see that $P(q) = -100q^2 + 5250q - 2500$.

(d) Find *all* break even points (if there are any), then briefly explain how you found the break even points.

Kenny's break even quantities are $q = 0.48$ and $q = 52.02$

EXPLANATION: To find break even points, first, we graph the profit function. It's graph is a parabola opening downward, so we have two break even points. To find these we goal seek $P(q)$ to be zero. Since the first break even point is near zero, we can goal seek with an intially blank quantity cell. The second break even point is near 60, so we enter 60 in the quantity cell before goal seeking a second time.

3. (12 points) Dunder Mifflin has recorded the following profit data from their Scranton, PA office:

Reams of paper sold (in thousands)	10	25	40	100
Profit ($)	-$120,000	-$10,000	$45,000	$25,000

(a) Find the quadratic (second order polynomial) which best models their profit function.

Enter the data in Excel, graph it, and add a second order polynomial trendline.
We get $P(q) = -66.47q^2 + 8880.8q - 198724$.

(b) Use your model to find $MP(15) = P(15) - P(14) = (-\$80,467.75) - (-\$87,420.92) = \$6,953.17$.

(c) Suppose that you know the following: $MP(67) > 0$, $MP(68) < 0$, and $MP(67.3) = 0$. What happens if the Scranton branch sells 67,300 reams of paper? Why?

Marginal profit at q measures how much profit is earned from the q^{th} item. So $MP(67) > 0$ means that after selling 66,000 reams of paper the next 1,000 reams will earn profit. On the other hand $MP(68) < 0$ means that after selling 67,000 reams of paper the next 1,000 will lose money. The fact that $MP(67.3) = 0$ says that profit is not growing or shrinking when 67,300 reams of paper are sold – their profit was growing up to this point and now selling any more paper will start to eat into their profits! Thus the Scranton branch will maximize its profit by selling 67,300 reams of paper.

(d) Suppose Dunder Mifflin's Elmira branch found that $MP(70) = -1250$ (70 represents 70,000 reams of paper sold). What does this tell us? Can we conclude from this that the Elmira branch will lose money if they sell 70,000 reams of paper?

$MP(70) = -1250$ says that after selling 69,000 reams of paper, the Elmira branch will lose money on the next 1,000 reams. We **cannot** conclude that the Elmira branch will lose money if they sell 70,000 reams of paper. However, we **can** conclude that they will either make more money or not lose as much money if they stick to selling 69,000 reams instead of selling 70,000 reams.

4. (13 points) Let $f(x) = -3x^2 + 5x + 1$.

(a) Fill out the following table:

$h =$	1	0.01	0.0001
$\dfrac{f(1+h) - f(1)}{h} =$	-4	-1.03	-1.0003

(b) Use your table to approximate $f'(1) = -1$

(c) Recall that $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$. Use the limit definition of the derivative to find $f'(x)$.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(-3(x+h)^2 + 5(x+h) + 1) - (-3x^2 + 5x + 1)}{h} \\
&= \lim_{h\to 0} \frac{-3x^2 - 6xh - 3h^2 + 5x + 5h + 1 + 3x^2 - 5x - 1}{h} \\
&= \lim_{h\to 0} \frac{-6xh - 3h^2 + 5h}{h} = \lim_{h\to 0} \frac{h(-6x - 3h + 5)}{h} \\
&= \lim_{h\to 0} -6x - 3h + 5 = -6x + 5
\end{aligned}$$

5. (42 points) Given $f(x)$, find $f'(x)$.

(a) $f(x) = \dfrac{x^2}{\sqrt{x}} + 5e^x = x^2 x^{-1/2} + 5e^x = x^{3/2} + 5e^x \Longrightarrow f'(x) = (3/2)x^{1/2} + 5e^x$

(b) $f(x) = \ln\left(\dfrac{x^3}{e^{-2x}}\right) = \ln(x^3) - \ln(e^{-2x}) = 3\ln(x) - (-2x) = 3\ln(x) + 2x \Longrightarrow f'(x) = \dfrac{3}{x} + 2$

(c) $f(x) = x^3 \ln(x) + 4x - 6 \Longrightarrow f'(x) = 3x^2 \ln(x) + x^3 \dfrac{1}{x} + 4 = 3x^2 \ln(x) + x^2 + 4$

(d) $f(x) = \dfrac{e^x + x^3}{x^2 + 1} \Longrightarrow f'(x) = \dfrac{(e^x + 3x^2)(x^2+1) - (e^x + x^3)(2x)}{(x^2+1)^2}$

$$= \frac{x^2e^x + 3x^4 + e^x + 3x^2 - 2xe^x - 2x^4}{(x^2+1)^2} = \frac{(x^2 - 2x + 1)e^x + x^4 + 3x^2}{(x^2+1)^2} = \frac{(x-1)^2 e^x + x^2(x^2+3)}{(x^2+1)^2}$$

(e) $f(x) = \sqrt{x^3 - 2x^2} = (x^3 - 2x^2)^{1/2} \Longrightarrow f'(x) = \dfrac{1}{2}(x^3 - 2x^2)^{-1/2}(3x^2 - 4x) = \dfrac{x(3x-4)}{2\sqrt{x^3 - 2x^2}}$

(f) $f(x) = x^2 e^{-2x} \ln(x^2+3) \Longrightarrow f'(x) = 2xe^{-2x}\ln(x^2+3) + x^2e^{-2x}(-2)\ln(x^2+3) + x^2e^{-2x}\dfrac{1}{x^2+3}(2x)$

6. (8 points) Find the equation of the line tangent to the graph of $y = 2x + e^x$ at the point where $x = 0$.

If $x = 0$, then $y = 2(0) + e^0 = 1$. So the tangent line passes through the point $(x, y) = (0, 1)$. We know that the derivative is a formula for the slopes of tangent lines. Since $y' = 2 + e^x$, we see that $y'|_{x=0} = 2 + e^0 = 3$ so that the slope of our tangent line is $m = 3$. Therefore, the equation of our tangent is $y - 1 = 3(x - 0)$ which simplifies to $y = 3x + 1$.