Math 1030 **Final Exam — Part I.** **August 7^{th}, 2009**

ANSWER KEY

1. (8 points) Find the derivative.

(a) $f(x) = \frac{x^2+1}{e^x-3x+2} + x^2\ln(x)$ Use the quotient rule and product rule.

$$f'(x) = \frac{2x(e^x-3x+2) - (x^2+1)(e^x-3)}{(e^x-3x+2)^2} + 2x\ln(x) + x^2\frac{1}{x}$$

(b) $g(x) = \sqrt{5x+2} + e^{x^3-6} = (5x+2)^{1/2} + e^{x^3-6}$ Use the chain rule.

$$g'(x) = \frac{1}{2}(5x+2)^{-1/2}(5) + e^{x^3-6}(3x^2)$$

2. (12 points) An integral problem.

(a) $\displaystyle\int 2 + \frac{3}{x} + \frac{4}{x^2} + \sqrt{x}\,dx = \int 2 + \frac{3}{x} + 4x^{-2} + x^{1/2}\,dx = 2x + 3\ln|x| + 4\frac{x^{-1}}{-1} + \frac{x^{3/2}}{3/2} + C$

Answer: $2x + 3\ln|x| - \frac{4}{x} + \frac{2}{3}\sqrt{x^3} + C$

(b) $\displaystyle\int (3x^2+2)e^{x^3+2x} + \frac{x+1}{x^2+2x-6}\,dx$

Split the integral up into two pieces. Use a substitution on each piece. The first piece we use: $u = x^3+2x$ so that $du = (3x^2+2)\,dx$. On the second part we use: $u = x^2+2x-6$ so that $du = (2x+2)\,dx = 2(x+1)\,dx$ and so $(1/2)\,du = (x+1)\,dx$

$$\int (3x^2+2)e^{x^3+2x}\,dx = \int e^u\,du = e^u + C = e^{x^3+2x} + C$$

$$\int \frac{x+1}{x^2+2x-6}\,dx = \int \frac{(1/2)\,du}{u} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+2x-6| + C$$

Answer: $e^{x^3+2x} + \frac{1}{2}\ln|x^2+2x-6| + C$

(c) Find $g(x)$ if $g'(x) = 5x^4 - \frac{1}{x}$ and $g(1) = 2$.

$g(x) = \int g'(x)\,dx = \displaystyle\int 5x^4 - \frac{1}{x}\,dx = x^5 - \ln|x| + C$ and we know that $2 = g(1) = 1^5 - \ln|1| + C = 1 - 0 + C$ so $C = 1$.

Answer: $g(x) = x^5 + \ln|x| + 1$