

Name: _____

Be sure to show your work!

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) + C \quad \int \sec^2(x) dx = \tan(x) + C$$

$$\tan^2(x) + 1 = \sec^2(x) \quad \sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\sqrt{a^2 - x^2} \leftrightarrow x = a \cdot \cos(\theta)$$

1. (____/16 points) Volumes.

- (a) Consider the solid obtained by rotating the region bounded by $y = x^2 - 2$ and $y = 3x - 2$ about the axis $y = 7$. Find an integral which computes this volume, but **do not evaluate your integral**.

- (b) Consider the solid obtained by rotating the region (in the first quadrant) bounded by $y = x^2$, $x = 0$, and $y = 4$ about the y -axis (i.e. $x = 0$). Find an integral which computes this volume, but **do not evaluate your integral**.

2. (____/10 points) Kyle developed a mystery liquid whose density is 20 lbs/ft^3 . Currently Kyle's liquid completely fills a cylindrical tank whose radius is 3 feet and height is 10 feet (the flat sides of the tank are parallel with the ground). Find an integral which computes the amount of work required to pump Kyle's liquid into a vat located 5 feet above the top of the tank. **Do not evaluate your integral.** Sketch a picture to indicate how you arrived at your integral.

3. (____/15 points) Differential Equations.

(a) Is $y = \sin(x)$ a solution of $y'' + 2y' + y = 2\cos(x)$? Why or why not?

(b) Solve the following initial value problem: $y' = \frac{2x+1}{2y}$ where $y(0) = -3$.

4. (____/7 points) Write down the “forms” which we would use to perform the partial fraction decomposition of $\frac{9x^5 + 2x^3 - 3x^2 - 5}{x(x-2)^3(x^2+1)(x^2+4)^2}$

5. (____/26 points) Integrate.

(a) $\int \frac{-x^2 - 3}{x(x^2 + 1)} dx$

(b) $\int x^2 \sin(5x) dx$

6. (____/26 points) Integrate.

(a) $\int e^{2x} \cos(x) dx$

(b) $\int \frac{\sqrt{x^2 - 1}}{x} dx$

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$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) + C \quad \int \sec^2(x) dx = \tan(x) + C$$

$$\tan^2(x) + 1 = \sec^2(x) \quad \sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\sqrt{a^2 - x^2} \leftrightarrow x = a \cdot \cos(\theta)$$

1. (_____/16 points) Volumes.

- (a) Consider the solid obtained by rotating the region bounded by $y = x^2 + 2$ and $y = 6$ about the axis $y = 1$. Find an integral which computes this volume, but **do not evaluate your integral**.

- (b) Consider the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 9$ about the y -axis (i.e. $x = 0$). Find an integral which computes this volume, but **do not evaluate your integral**.

2. (____/10 points) Kyle developed a mystery liquid whose density is 30 lbs/ft^3 . Currently Kyle's liquid completely fills a rectangular tank whose cross-sections are squares (2 feet by 2 feet) and height is 5 feet. Find an integral which computes the amount of work required to pump Kyle's liquid into a vat located 3 feet above the top of the tank. **Do not evaluate your integral.** Sketch a picture to indicate how you arrived at your integral.

3. (____/15 points) Differential Equations.

(a) Is $y = x^3$ a solution of $y'' + 2y' + y = 5$? Why or why not?

(b) Solve the following initial value problem: $y' = y \cos(x)$ where $y(0) = 2$.

4. (____/7 points) Write down the “forms” which we would use to perform the partial fraction decomposition of $\frac{3x^6 - 2x^4 + x^3 - 1}{x^2(x + 5)^3(x^2 + 9)}$

5. (____/26 points) Integrate.

(a) $\int x \ln(x) dx$

(b) $\int \frac{x^2 + 3x - 2}{(x - 1)(x^2 + 1)} dx$

6. (____/26 points) Integrate.

(a) $\int \tan^3(x) \sec^4(x) dx$

(b) $\int \sqrt{4 - x^2} dx$