Math 1120-101 # Test #3 **November 10$^{\text{th}}$, 2010**

Name: ______________________________ **Be sure to show your work!**

$$|f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n+1)!}|x - x_0|^{n+1} \qquad \frac{1}{\sqrt{2\pi}s}e^{-\frac{(x-m)^2}{2s^2}}$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx \qquad a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx \qquad b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,dx$$

1. (____/24 points) Taylor Polynomials

(a) Let $f(x) = x^3 + 2x + 3$. Find the 4$^{\text{th}}$-order Taylor polynomial, $P_4(x)$, for $f(x)$ centered at $x_0 = -1$.

(b) Find a bound for $|g(x) - P_3(x)|$ where $g(x) = 3\cos(x)$, $P_3(x)$ is the 3$^{\text{rd}}$-order MacLaurin polynomial for $g(x)$, and $0 \leq x \leq 2$.

(c) The 50$^{\text{th}}$-order Taylor polynomial centered at $x_0 = 2$ of some function $h(x)$ is $P_{50}(x) = 6 + 3(x-2)^2 - 4(x-2)^5 + 5(x-2)^6 - 2(x-2)^{50}$.

$h(2) =$ ______________________________

$h^{(45)}(2) =$ ______________________________

2. (____/15 points) Fourier Polynomials

(a) Let $f(x) = \begin{cases} 1 & x < 0 \\ -1 & x \geq 0 \end{cases}$

Find the 1^{st}-order Fourier polynomial for $f(x)$.

(b) $g(x) = \sin^2(x) + 3\sin(x) = \frac{1}{2} - \frac{1}{2}\cos(2x) + 3\sin(x)$

$\int_{-\pi}^{\pi} g(x)\,dx =$ ____________________________

$\int_{-\pi}^{\pi} g(x)\cos(x)\,dx =$ ____________________________

$\int_{-\pi}^{\pi} g(x)\sin(x)\,dx =$ ____________________________

3. (____/16 points) An Improper Problem.

(a) Let $f(x) = e^{-2x}$ when $x \geq 0$ and $f(x) = 0$ when $x < 0$. Is $f(x)$ a probability distribution? Why or why not?

(b) Does $\int_{-\infty}^{0} \frac{1}{x^2}\, dx$ converge? If so, what does it converge to? If not, why not?

4. (____/16 points) Converge or Diverge?

Determine whether the following integrals converge or diverge. If they converge, you do **not** need to find what they converge to. If you use a comparison or other test, **SHOW YOUR WORK**.

(a) Does $\int_1^\infty \frac{\sin(x)}{x^2+1}\,dx$ converge or diverge? **Note: I've changed the lower bound from 0 to 1 to avoid an unintended technical difficulty.**

WARNING: $\sin(x)$ is not always positive.

(b) Does $\int_2^\infty \frac{x^2+6x+2}{x^3-x-3}\,dx$ converge or diverge?

5. (____/14 points) Probably a good problem.

(a) Suppose that the average weight of an incoming male student is 165 lbs. and that the weights of incoming male students are normally distributed with a standard deviation of 5. Write down an integral which computes the probability that an incoming male student weighs less than 140 lbs. Then convert your integral into an integral of the standard normal distribution.

(b) Given the same set-up as part (a), interpret the following integral as a probability.
Hint: First, convert from the standard normal back to the original weight distribution.

$$\int_{-3}^{7} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$$

Answer:
This integral computes the probability that an incoming male student...

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6. (____/15 points) Write the first 3 terms of each of the following sequences. If the sequence converges, explain why it converges and find its limit. If the sequence diverges, explain why it does not converge.

(a) $\left\{ \dfrac{3k^2 + (-1)^k}{k^2 + 1} \right\}_{k=0}^{\infty}$

(b) $\{\sin(k^2)\}_{k=0}^{\infty}$

(c) $\left\{ \dfrac{(-1)^k}{(k!)^2} \right\}_{k=1}^{\infty}$

Math 1120-102 **Test #3** **November 10th, 2010**

Name: ______________________________ **Be sure to show your work!**

$$|f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n+1)!}|x - x_0|^{n+1} \qquad \frac{1}{\sqrt{2\pi}s}e^{-\frac{(x-m)^2}{2s^2}}$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx \qquad a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx \qquad b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,dx$$

1. (____/24 points) Taylor Polynomials

(a) Let $f(x) = -2x^3 + 3x^2 + 1$. Find the 4th-order Taylor polynomial, $P_4(x)$, for $f(x)$ centered at $x_0 = 1$.

(b) Find a bound for $|g(x) - P_2(x)|$ where $g(x) = 3e^{-x}$, $P_2(x)$ is the 2nd-order MacLaurin polynomial for $g(x)$, and $0 \leq x \leq 2$.

(c) The 100th-order Taylor polynomial centered at $x_0 = -3$ of some function $h(x)$ is $P_{100}(x) = -2(x+3)^3 + 3(x+3)^6 - 2(x+3)^{20} + 5(x+3)^{77} - 2(x+3)^{100}$.

$h^{(40)}(-3) =$ ______________________________

$h'''(-3) =$ ______________________________

2. (____/15 points) Fourier Polynomials

(a) Let $f(x) = \begin{cases} 1 & x < 0 \\ -1 & x \geq 0 \end{cases}$

Find the 1$^{\text{st}}$-order Fourier polynomial for $f(x)$.

(b) The 3$^{\text{rd}}$-order Fourier polynomial for some function $g(x)$ is $q_3(x) = 5+2\cos(2x)+\sin(x)-4\sin(3x)$.

$\int_{-\pi}^{\pi} g(x)\cos(x)\,dx =$ ____________________________

$\int_{-\pi}^{\pi} g(x)\sin(3x)\,dx =$ ____________________________

Circle One:
$g(x)$ is EVEN ODD BOTH NEITHER CAN'T TELL

Why? Explain your answer.

3. (____/16 points) An Improper Problem.

(a) Let $f(x) = e^{-x}$ when $x \geq 0$ and $f(x) = 0$ when $x < 0$. Is $f(x)$ a probability distribution? Why or why not?

(b) Does $\displaystyle\int_0^\infty \frac{1}{x^2}\,dx$ converge? If so, what does it converge to? If not, why not?

4. (____/16 points) Converge or Diverge?

Determine whether the following integrals converge or diverge. If they converge, you do **not** need to find what they converge to. If you use a comparison or other test, **SHOW YOUR WORK**.

(a) Does $\displaystyle\int_1^\infty \frac{x^2\sin(x)}{x^5+4}\,dx$ converge or diverge? **Note: I've changed the lower bound from 0 to 1 to avoid an unintended technical difficulty.**

WARNING: $\sin(x)$ is not always positive.

(b) Does $\displaystyle\int_2^\infty \frac{x+\cos^2(x)}{x-1}\,dx$ converge or diverge?

5. (____/14 points) Probably a good problem.

(a) Suppose that the average height of Boone residents is 68 inches (5 foot 8 inches). In addition suppose that these heights are normally distributed with a standard deviation of 2. Write down an integral which computes the probability that a resident of Boone is between 66 and 72 inches tall. Then convert your integral into an integral of the standard normal distribution.

(b) Given the same set-up as part (a), interpret the following integral as a probability.
Hint: First, convert from the standard normal back to the original height distribution.

$$\int_{-\infty}^{-4} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$$

Answer:
This integral computes the probability that a resident of Boone...

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6. (____/15 points) **Write the first 3 terms of each of the following sequences.** If the sequence converges, explain why it converges and find its limit. If the sequence diverges, explain why it does not converge.

(a) $\left\{ \dfrac{(-1)^k k^2}{k^4+1} \right\}_{k=0}^{\infty}$

(b) $\left\{ \cos\left(\dfrac{1}{k!}\right) \right\}_{k=2}^{\infty}$

(c) $\left\{ \dfrac{k+2}{e^{-k}} \right\}_{k=1}^{\infty}$