

Name: _____

Be sure to show your work!

- Double angle: $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$, $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, $\sin(2x) = 2\sin(x)\cos(x)$
- Angle sum and difference: $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$, $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$,
 $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$, $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
- Product to sum: $2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y)$, $2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y)$,
 $2\sin(x)\cos(y) = \sin(x-y) + \sin(x+y)$

1. (25 points) Pythagorean Theorem

- (a) Write down the Pythagorean Theorem trigonometric identity relating sine and cosine. Then write down the other version relating tangent and secant.

Identity #1: _____

Identity #2: _____

- (b) In each of the following integrals, use the proper (inverse) trigonometric substitution to simplify the integral. **Do NOT** integrate. Just substitute and **simplify**.

$$\bullet \int \frac{dx}{x^2\sqrt{x^2+9}}$$

$$\bullet \int \frac{\sqrt{x^2-4}}{x} dx$$

$$\bullet \int_0^5 \sqrt{25-x^2} dx$$

- (c) Given $x = 3\sin(\theta)$, rewrite $7\theta - 4\tan(\theta)$ in terms of x .

2. (8 points) Write down the “forms” we would use to find the partial fraction decomposition of $\frac{3x^5 - x^4 + 9x^2 + 8}{x(x+6)^3(x^2+x+3)^2}$.

3. (15 points) Complete the Square

- (a) To integrate $\int \frac{6x}{\sqrt{1+3x-x^2}} dx$ we would need to split the integral into a u -substitution integral and an integral where we complete the square in the radical. Split this integral into those two pieces. **Do NOT** integrate.

$$\int \frac{6x}{\sqrt{1+3x-x^2}} dx = \underline{\hspace{10em}} + \underline{\hspace{10em}}$$

- (b) Compute $\int \frac{3}{x^2-4x+13} dx$

4. (18 points) Integrate!

(a) $\int x^2 e^{4x} dx$

(b) $\int \sin^3(x) \cos^4(x) dx$

(c) $\int_0^1 \arctan(x) dx$

5. (34 points) Integrate!

(a) $\int e^x \sin(3x) dx$

(b) $\int \tan^5(x) \sec^4(x) dx$

(c) $\int \sqrt{4-x^2} dx$

(d) $\int \frac{2x^2 + 7x + 4}{x(x+1)^2} dx$

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1. (25 points) Pythagorean Theorem

- (a) Write down the Pythagorean Theorem trigonometric identity relating sine and cosine. Then write down the other version relating tangent and secant.

Identity #1: _____

Identity #2: _____

- (b) In each of the following integrals, use the proper (inverse) trigonometric substitution to simplify the integral. **Do NOT** integrate. Just substitute and **simplify**.

- $\int \frac{dx}{x^2\sqrt{x^2 + 4}}$

- $\int \frac{\sqrt{x^2 - 9}}{x} dx$

- $\int_0^4 \sqrt{16 - x^2} dx$

- (c) Given $x = 5\sin(\theta)$, rewrite $4\theta - 7\tan(\theta)$ in terms of x .

- 2. (8 points)** Write down the “forms” we would use to find the partial fraction decomposition of $\frac{3x^5 - x^4 + 9x^2 + 8}{x^3(x + 7)(x^2 + x + 11)^2}$.

3. (15 points) Complete the Square

- (a) To integrate $\int \frac{10x}{\sqrt{1+4x-x^2}} dx$ we would need to split the integral into a u -substitution integral and an integral where we complete the square in the radical. Split this integral into those two pieces. **Do NOT** integrate.

$$\int \frac{10x}{\sqrt{1+4x-x^2}} dx = \underline{\hspace{10em}} + \underline{\hspace{10em}}$$

- (b) Compute $\int \frac{2}{x^2-6x+13} dx$

4. (18 points) Integrate!

(a) $\int x^2 e^{-x} dx$

(b) $\int \sin^4(x) \cos^3(x) dx$

(c) $\int_0^1 \arctan(x) dx$

5. (34 points) Integrate!

(a) $\int e^x \cos(3x) dx$

(b) $\int \tan^3(x) \sec^6(x) dx$

(c) $\int \sqrt{9 - x^2} dx$

(d) $\int \frac{3x^2 + 1}{x(x - 1)^2} dx$