Math 1120 **Test #3** **April 3rd, 2009**

Name: _____ **Be sure to show your work!**

$$|f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n+1)!}|x - x_0|^{n+1} \qquad \frac{1}{\sqrt{2\pi}s}e^{-\frac{(x-m)^2}{2s^2}}$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx \qquad a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx \qquad b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,dx$$

1. (____/24 points) Taylor Polynomials

(a) Let $f(x) = \ln(x)$. Find the 2^{nd}-order Taylor polynomial, $P_2(x)$, for $f(x)$ centered at $x_0 = 1$.

(b) Find the maximum error $|f(x) - P_2(x)|$ allowed by Taylor's error estimate if $1 \leq x \leq 2$.

(c) Suppose that the MacLaurin polynomial of some function $g(x)$ is $P_{2n+1}(x) = \sum_{k=0}^{n} \frac{(-1)^k}{2k+1}x^{2k+1}$

$g^{(100)}(0) =$ _____

$g'''(0) =$ _____

2. (____/15 points) Fourier Polynomials

(a) Let $f(x) = x$. Find the 1^{st}-order Fourier polynomial for $f(x)$.

(b) Suppose the 2^{nd}-order Fourier polynomial for $g(x)$ is $q_2(x) = 1 + 3\cos(x) - 2\sin(2x) + 5\cos(2x)$.

$\int_{-\pi}^{\pi} g(x)\,dx =$ _____

$\int_{-\pi}^{\pi} g(x)\cos(2x)\,dx =$ _____

$\int_{-\pi}^{\pi} g(x)\sin(x)\,dx =$ _____

3. (____/16 points) An Improper Problem.

(a) Let $f(x) = \begin{cases} x^{-2} & x \geq 1 \\ 0 & x < 1 \end{cases}$. Is $f(x)$ a probability distribution? Why or why not?

(b) Does $\int_{-\infty}^{0} xe^{(-x^2)}\,dx$ converge? If so, what does it converge to? If not, why not?

4. (____/16 points) Converge or Diverge?

Determine whether the following integrals converge or diverge. If they converge, you do **not** need to find what they converge to. If you use a comparison test, **SHOW YOUR WORK**.

(a) Does $\displaystyle\int_{-1}^{1} \frac{\ln|x|}{x}\,dx$ converge or diverge?

(b) Does $\displaystyle\int_{1}^{\infty} \frac{2+\cos(x)}{x^5}\,dx$ converge or diverge?

5. (____/14 points) The average annual precipitation on Grandfather mountain is about 62 inches with a standard deviation of about 10 inches. Assume annual precipitation is distributed normally.

(a) Write down an integral which computes the probability that Grandfather mountain will have over 77 inches of precipitation (in 1 year). Then convert your integral into an an integral of the standard normal distribution.

(b) Write down an integral which computes the probability that Grandfather mountain will have between 52 and 82 inches of precipitation (in 1 year). Then convert your integral into an an integral of the standard normal distribution.

6. (____/15 points) Write the first 3 terms of each of the following sequences. If the sequence converges, explain why it converges and find its limit. If the sequence diverges, explain why it does not converge.

(a) $\left\{\cos(e^{-k})\right\}_{k=1}^{\infty}$

(b) $\left\{\dfrac{(-1)^k(k-1)!}{k!}\right\}_{k=1}^{\infty}$

(c) $\left\{\ln\left(\dfrac{1}{k}\right)\right\}_{k=1}^{\infty}$