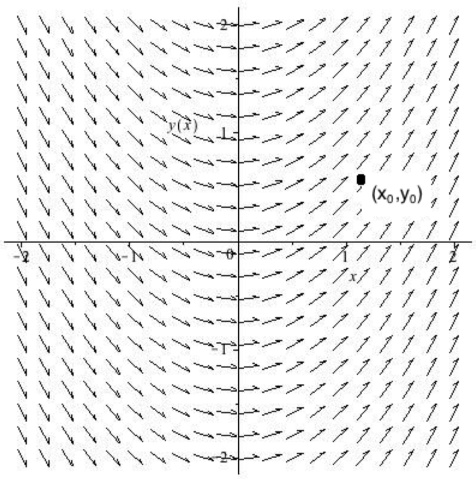
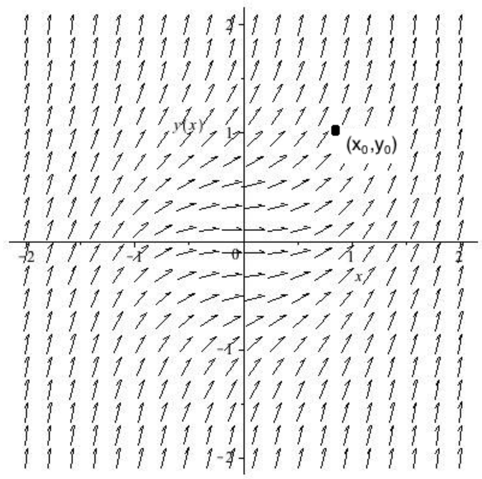
Math 1120-101 **Final Exam** **May 8^{th}, 2014**

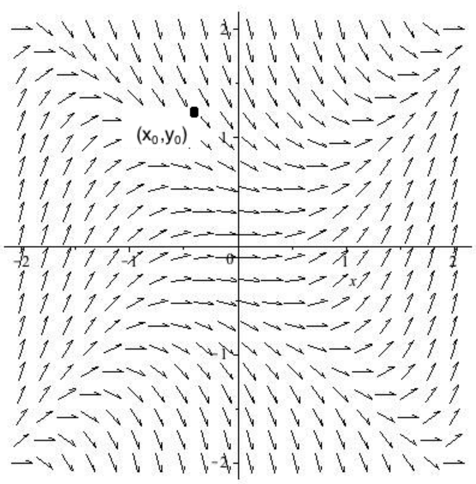
Name: ______________________________ **Be sure to show your work!**

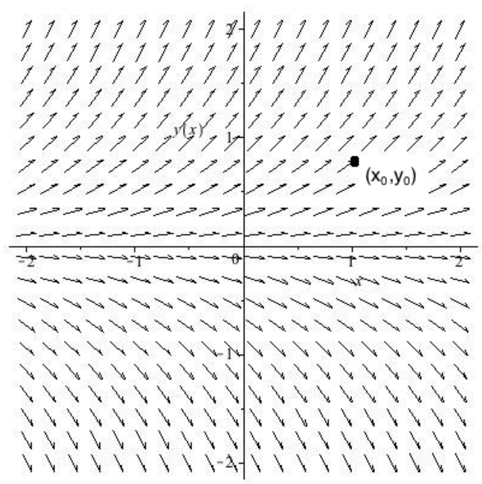
1. (11 points) Slope fields and Euler's method.

(a) Sketch the solution through the marked point (x_0, y_0) on each slope field. Then label each slope field with its equation (A, B, C, D, or E). *Note:* Obviously the slope field of one of the equations is not graphed below.









Possible differential equations:

A $y' = x$ B $y' = x^2 + y^2$ C $y' = -x^2 - y^2$ D $y' = y$ E $y' = x^2 - y^2$

(b) Use Euler's method to approximate $y(5)$ where $y(x)$ is the solution of $y' = x^2 + y$ and $y(-1) = 1$. Use $n = 2$ steps.

2. (12 points) Differential Equations.

(a) Is $y = \sin(x)$ a solution of the differential equation $y'' + 3y' + y = 3\cos(x)$? Why or why not?

(b) Find the general solution of $xy' = \dfrac{x + 2x^2}{2y}$.

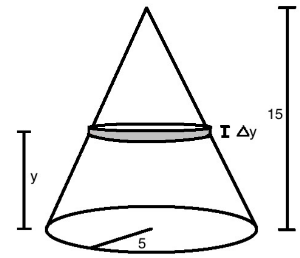
(c) I pour a cup of coffee. Initially the temperature of the coffee is 160°F. My cup then sits in a 60°F room and after 2 hours the coffee is 80°F. Use Newton's law of cooling to find a differential equation governing the temperature of my coffee. Then solve this equation and find a formula for $T(t)$ (the temperature of the coffee in °F, t hours after the coffee is poured). [Fill in the blanks below. Simplify as much as possible.]

$T(t) =$ ____

$T(4) =$ ____________ °F

$\lim_{t \to \infty} T(t) =$ ____________ °F

3. (12 points) Volumes



(a) Consider a cone which is 15 units tall and whose circle base has a radius of 5 units. Slice this cone horizontally. Let y denote the distance from the **bottom** of the cone.

i. Find a formula for the volume of a slice of height Δy at height y.

ii. Using the previous result, find a definite integral which computes the volume of this cone. Do **not** evaluate this integral.

(b) Set up an integral which computes the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = 8 - x^2$ about the x-axis. You **do not** need to evaluate this integral.

(c) Set up an integral which computes the volume of the solid obtained by taking the region bounded by $y = \sqrt{x}$, the y-axis, and $y = 2$ then rotating about the y-axis. Again, you do **not** need to evaluate this integral.
WARNING: I said rotate about the y-axis...this is a vertical axis.

4. (11 points) Evaluate the integrals.

(a) $\displaystyle\int_{-1}^{1} \frac{x+1}{x^2+2x+3}\, dx$

(b) $\displaystyle\int x^2 e^{2x}\, dx$

5. (11 points) Evaluate the integrals.

(a) $\displaystyle\int \frac{2x^2 + 3x + 2}{x^3 + x}\, dx$ **Hint:** Partial fractions.

(b) $\displaystyle\int \cos^5(x)\, dx$

6. (11 points) Evaluate the integrals.

(a) $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx$ You don't need to simplify your answer.

(b) $\displaystyle\int e^{-x}\sin(x)\,dx$

7. (10 points) Improper integrals.

(a) Does $\int_0^2 (x-1)^{-2}\,dx$ converge or diverge? If the integral converges, find what it converges to.

(b) Use a convergence test (or tests) to determine if the following integral converges or diverges:
$\int_1^{\infty} \frac{x^2\cos^4(x)}{x^5+3x^2+9}\,dx$

8. (11 points) Power Series

(a) Consider the power series $\sum_{n=1}^{\infty} \frac{(x-5)^n}{n^3 2^n}$. Find its radius **and** interval of convergence. Show your work – indicate which tests you used to determine your answers.

(b) Suppose $f(x) = \sum_{n=0}^{\infty} a_n (x+3)^n$ is a power series which converges when $x = 0$ and diverges when $x = 2$. For each of the following values of x indicate whether $f(x)$ converges, diverges, or we need more information.

(i) $x = 6$ Converges / Diverges / Need More Information

(ii) $x = -6$ Converges / Diverges / Need More Information

(iii) $x = -5$ Converges / Diverges / Need More Information

(iv) $x = 1$ Converges / Diverges / Need More Information

9. (11 points) Finding Series Expansions

(a) Find the first 5 terms (i.e. $???+???x+???x^2+???x^3+???x^4...$) of the MacLaurin series of $f(x) = \cos^2(x)$ **Hint:** Use a trigonometric identity first.

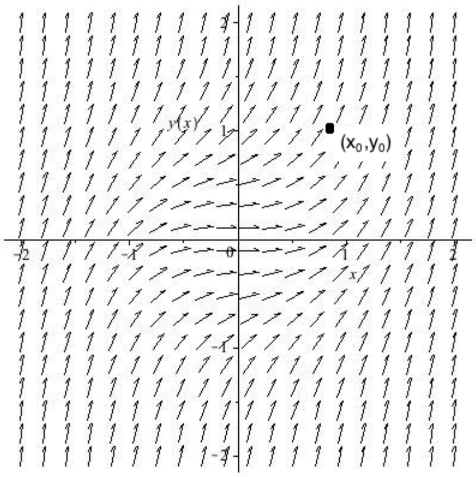
(b) Find a formula for the Taylor series of $f(x) = \ln(x)$ centered at $x_0 = 1$.
Hint: Compute the first few terms and look for a pattern OR notice that $\ln(x) = \ln(1 + (x - 1))$

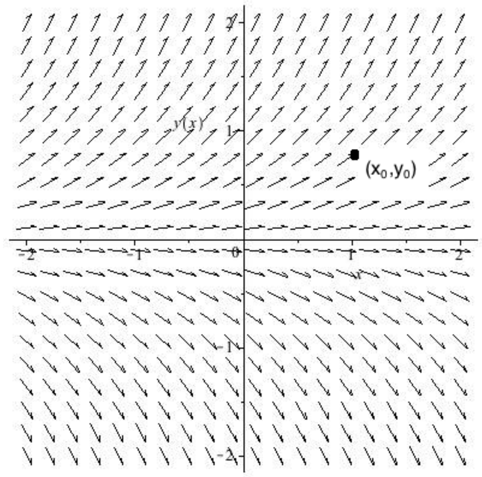
Math 1120-108 # Final Exam **May 7$^{\text{th}}$, 2014**

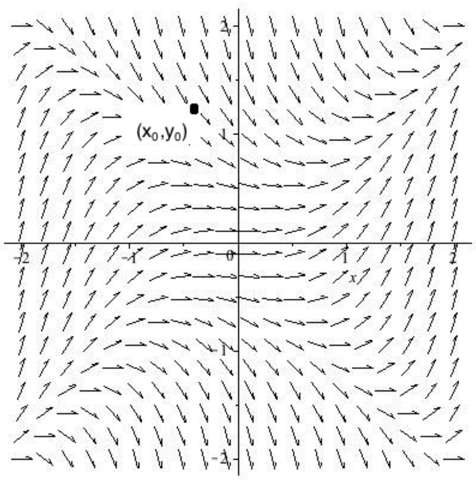
Name: ________________________________ **Be sure to show your work!**

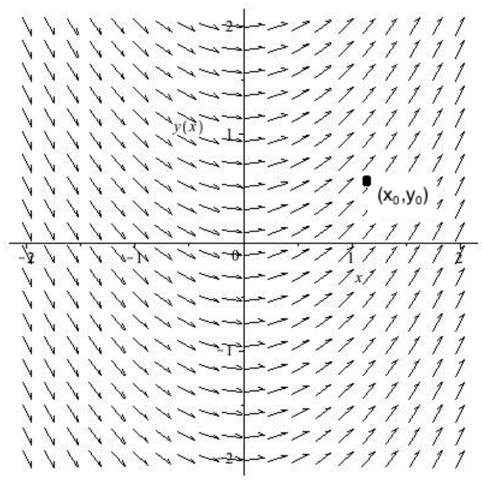
1. (11 points) Slope fields and Euler's method.

(a) Sketch the solution through the marked point (x_0, y_0) on each slope field. Then label each slope field with its equation (A, B, C, D, or E). *Note:* Obviously the slope field of one of the equations is not graphed below.









Possible differential equations:

A $y' = x$ B $y' = x^2 + y^2$ C $y' = -x^2 - y^2$ D $y' = y$ E $y' = x^2 - y^2$

(b) Use Euler's method to approximate $y(4)$ where $y(x)$ is the solution of $y' = 2x^2 - y$ and $y(-2) = 6$. Use $n = 2$ steps.

2. (12 points) Differential Equations.

(a) Is $y = x^3$ a solution of the differential equation $x^2 y'' - 5y = 1$? Why or why not?

(b) Find the general solution of $y' = -2xy$.

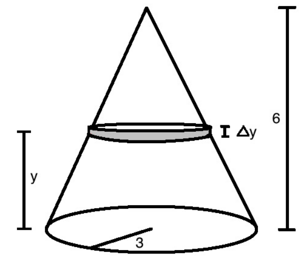
(c) I pour a cup of coffee. Initially the temperature of the coffee is 150°F. My cup then sits in a 70°F room and after 3 hours the coffee is 80°F. Use Newton's law of cooling to find a differential equation governing the temperature of my coffee. Then solve this equation and find a formula for $T(t)$ (the temperature of the coffee in °F, t hours after the coffee is poured). [Fill in the blanks below. Simplify as much as possible.]

$T(t) =$ ____

$T(6) =$ ______________ °F

$\lim_{t \to \infty} T(t) =$ ______________ °F

3. (12 points) Volumes



(a) Consider a cone which is 6 units tall and whose circle base has a radius of 3 units. Slice this cone horizontally. Let y denote the distance from the **bottom** of the cone.

i. Find a formula for the volume of a slice of height Δy at height y.

ii. Using the previous result, find a definite integral which computes the volume of this cone. Do **not** evaluate this integral.

(b) Set up an integral which computes the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = x + 2$ about the x-axis. You **do not** need to evaluate this integral.

(c) Set up an integral which computes the volume of the solid obtained by taking the region bounded by $y = x^3$, the y-axis, and $y = 8$ then rotating about the y-axis. Again, you do **not** need to evaluate this integral.
WARNING: I said rotate about the y-axis...this is a vertical axis.

4. (11 points) Evaluate the integrals.

(a) $\displaystyle\int_0^{\sqrt{\pi/2}} 2x\cos(x^2)e^{\sin(x^2)}\,dx$

(b) $\displaystyle\int x^2\cos(x)\,dx$

5. (11 points) Evaluate the integrals.

(a) $\displaystyle\int \frac{4x^2+3x+2}{x^3+x^2}\,dx$ **Hint:** Partial fractions.

(b) $\displaystyle\int \tan(x)\sec^4(x)\,dx$

6. (11 points) Evaluate the integrals.

(a) $\int x\sqrt{1-x^2}\,dx$ Simplify your answer.

(b) $\int e^{2x}\cos(x)\,dx$

7. (10 points) Improper integrals.

(a) Does $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx$ converge or diverge? If the integral converges, find what it converges to.

(b) Use a convergence test (or tests) to determine if the following integral converges or diverges:
$\int_{2}^{\infty} \frac{xe^x + 3\sqrt{x} + 5}{x-1}\, dx$

8. (11 points) Power Series

(a) Consider the power series $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n3^n}$. Find its radius **and** interval of convergence.
Show your work – indicate which tests you used to determine your answers.

(b) Suppose $f(x) = \sum_{n=0}^{\infty} a_n(x-1)^n$ is a power series which converges when $x = 3$ and diverges when $x = 5$. For each of the following values of x indicate whether $f(x)$ converges, diverges, or we need more information.

(i) $x = 0$ Converges / Diverges / Need More Information

(ii) $x = -4$ Converges / Diverges / Need More Information

(iii) $x = 4$ Converges / Diverges / Need More Information

(iv) $x = -1$ Converges / Diverges / Need More Information

9. (11 points) Finding Series Expansions

(a) Find the first 3 terms (i.e. ???+???x+???x^2...) of the Taylor series of $f(x) = e^{x^2-1}$ centered at $x = 1$.

(b) Find a formula for the MacLaurin series of $f(x) = x^7 \sin(3x^2)$.