Math 1120-101 **Test #1** **February 10^{th}, 2014**

Name: ANSWER KEY **Be sure to show your work!**

$\sin^2(x) = \frac{1}{2}\Big(1-\cos(2x)\Big)$ $\frac{d}{dx}\Big[\tan(x)\Big] = \sec^2(x)$ $\frac{d}{dx}\Big[\text{asin}(x)\Big] = \frac{1}{\sqrt{1-x^2}}$

1. (30 points) Basic Integrals

(a) $\displaystyle\int x^2e^{3x}\,dx$

The natural way to approach this integral is via *integration by parts*. Seeing the "x^2" next to the exponential indicates we'll have to do by-parts twice. First, choose $u = x^2$ and $dv = e^{3x}\,dx$ (since we can integrate either part, we choose a u which becomes simpler when differentiated). Thus $du = 2x\,dx$ and $v = \frac{1}{3}e^{3x}$.

$$\int x^2e^{3x}\,dx = uv - \int v\,du = x^2\cdot\frac{1}{3}e^{3x} - \int\frac{1}{3}e^{3x}2x\,dx$$

Now let $u = -\frac{2}{3}x\,dx$ and $dv = e^{3x}\,dx$ so that $du = -\frac{2}{3}$ and $v = \frac{1}{3}e^{3x}$.

$$\int x^2e^{3x}\,dx = \cdots = \frac{1}{3}x^2e^{3x} - \frac{2}{3}x\cdot\frac{1}{3}e^{3x} - \int\frac{1}{3}e^{3x}\left(-\frac{2}{3}\right)\,dx = \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$$

Alternatively, this problem can be tackled using an "undetermined coefficients" technique. The derivatives of x^2e^{3x} fall into the pattern $(Ax^2+Bx+C)e^{3x}$ so we guess that the integral has an answer of this form. So if our answer is of the form $y = (Ax^2+Bx+C)e^{3x}$ + Constant, then $y' = (2Ax+B)e^{3x} + (Ax^2+Bx+C)3e^{3x} = (3Ax^2+(2A+3B)x+(B+3C))e^{3x}$. But we need that $y' = \frac{d}{dx}\int x^2e^{3x}\,dx = x^2e^{3x} = (1\cdot x^2+0x+0)e^{3x}$. Thus $3A = 1$, $2A+3B = 0$ and $B+3C = 0$. So $A = \frac{1}{3}$, $B = -\frac{2}{3}A = -\frac{2}{9}$, and $C = -\frac{1}{3}B = \frac{2}{27}$.

Answer: $\displaystyle\int x^2e^{3x}\,dx = \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$

(b) $\displaystyle\int_0^2\frac{x^2+1}{x^3+3x+1}\,dx$

This is a simple u-substitution problem. Notice that the derivative of the denominator is a multiple of the numerator. So we let $u = x^3+3x+1$ so that $du = (3x^2+3)\,dx = 3(x^2+1)\,dx$. Thus $\frac{1}{3}\,du = (x^2+1)\,dx$. Since this is a definite integral, we also need to change its bounds: $x = 0 \mapsto u = 0^3+3(0)+1 = 1$ and $x = 2 \mapsto u = 2^3+3(2)+1 = 15$. [Note: $\ln(1) = 0$.]

$$\int_0^2\frac{x^2+1}{x^3+3x+1}\,dx = \int_1^{15}\frac{1}{u}\cdot\frac{1}{3}\,du = \frac{1}{3}\ln|u|\Big|_1^{15} = \frac{1}{3}\ln(15) - \frac{1}{3}\ln(1) = \boxed{\frac{1}{3}\ln(15)}$$

(c) $\displaystyle\int_0^{\pi/2}\cos^3(x)\,dx$

Since we are integrating an *odd* power of cosine, we should use the identity "$\cos^2(x) = 1-\sin^2(x)$" to turn an even number of powers of cosine into sines leaving 1 cosine to serve as the derivative of our substitution.

$$\int_0^{\pi/2}\cos^3(x)\,dx = \int_0^{\pi/2}\cos^2(x)\cos(x)\,dx = \int_0^{\pi/2}(1-\sin^2(x))\cos(x)\,dx$$

Now we can make the substitution: $u = \sin(x)$ so that $du = \cos(x)\,dx$. Remember to change the bounds: $x = 0 \mapsto u = \sin(0) = 0$ and $x = \pi/2 \mapsto u = \sin(\pi/2) = 1$.

$$\cdots = \int_0^{\pi/2}(1-\sin^2(x))\cos(x)\,dx = \int_0^1(1-u^2)\,du = u - \frac{1}{3}u^3\Big|_0^1 = \left(1-\frac{1}{3}1^3\right) - \left(0-\frac{1}{3}0^3\right) = \boxed{\frac{2}{3}}$$

2. (30 points) More Integrals

(a) $\displaystyle\int \ln(x)\,dx$

Integration by parts will work here. Since we are trying to integrate $\ln(x)$, this needs to be our u. Thus we let $u = \ln(x)$ and $dv = dx$ so that $du = \frac{1}{x}\,dx$ and $v = x$.

$$\int \ln(x)\,dx = uv - \int v\,du = \ln(x)\cdot x - \int x\frac{1}{x}\,dx = x\ln(x) - \int 1\,dx = \boxed{x\ln(x) - x + C}$$

(b) $\displaystyle\int \sec^4(x)\,dx$

We should try the identity: $\sec^2(x) = 1 + \tan^2(x)$. Notice that the derivative of $\tan(x)$ is $\sec^2(x)$ so this works perfectly. Let $u = \tan(x)$ and $du = \sec^2(x)\,dx$.

$$= \int \sec^2(x)\cdot\sec^2(x)\,dx = \int (1+\tan^2(x))\sec^2(x)\,dx = \int (1+u^2)\,du = u + \frac{1}{3}u^3 + C = \boxed{\tan(x) + \frac{1}{3}\tan^3(x) + C}$$

(c) $\displaystyle\int e^x \sin(2x)\,dx$

This integral can be tackled by using integration by parts twice and then solving for the integral. Let $u = \sin(2x)$ and $dv = e^x\,dx$ so that $du = 2\cos(2x)\,dx$ and $v = e^x$.

$$I = \int e^x\sin(2x)\,dx = uv - \int v\,du = \sin(2x)e^x - \int e^x 2\cos(2x)\,dx$$

Now let $u = -2\cos(2x)$ and $dv = e^x\,dx$ so that $du = 4\sin(2x)\,dx$ and $v = e^x$.

$$I = \cdots = e^x\sin(2x) - 2\cos(2x)e^x - \int e^x 4\sin(2x)\,dx = e^x\sin(2x) - 2e^x\cos(2x) - 4I$$

This means that $5I = I + 4I = e^x\sin(2x) - 2e^x\cos(2x)$.

Answer: $\displaystyle\int e^x\sin(2x)\,dx = \frac{1}{5}e^x\sin(2x) - \frac{2}{5}e^x\cos(2x) + C$

Alternatively, we could use an "undetermined coefficients" technique to tackle this problem. Notice that the derivatives of our integrand have the form: $Ae^x\sin(2x) + Be^x\cos(2x)$. We make this guess $y = Ae^x\sin(2x) + Be^x\cos(2x)$ so that $y' = Ae^x\sin(2x) + Ae^x 2\cos(2x) + Be^x\cos(2x) + Be^x(-2\sin(2x)) = (A-2B)e^x\sin(2x) + (2A+B)e^x\cos(2x)$. This needs to match the integrand so we need $(A-2B)e^x\sin(2x) + (2A+B)e^x\cos(2x) = 1e^x\sin(2x) + 0e^x\cos(2x)$. Thus $A - 2B = 1$ and $2A + B = 0$. Multiply the second equation by 2 and add to the first equation and get $5A = 1$ so that $A = 1/5$. Thus the second equation says $2(1/5) + B = 0$ so that $B = -2/5$ giving us the same answer as before.

3. (8 points) Write down the "forms" we would use to find the partial fraction decomposition of…

$$\frac{9x^7 - 11x^5 + x^4 - 2x + 1}{x(x+5)^3(x^2+2x+6)^2} = \frac{A}{x} + \frac{B}{x+5} + \frac{C}{(x+5)^2} + \frac{D}{(x+5)^3} + \frac{Ex+F}{x^2+2x+6} + \frac{Gx+H}{(x^2+2x+6)^2}$$

4. (20 points) More Integrals

(a) $\displaystyle\int \frac{1}{x^2+2x+2}\,dx$

A u-substitution will not work here (a multiple of the derivative of the denominator does not appear as the numerator), so we should consider partial fractions. However, x^2+2x+2 does not factor (it has complex roots since $B^2 - 4AC = 2^2 - 4(1)(2) = -4 < 0$). Thus this must be the derivative of arc tangent in disguise. We should complete the square.

$$\int \frac{1}{x^2+2x+2}\,dx = \int \frac{1}{(x+1)^2+1}\,dx = \boxed{\arctan(x+1) + C}$$

(b) $\displaystyle\int \frac{2x^2-x-1}{x(x+1)^2}\,dx$

This is an obvious candidate for partial fractions.

$$\frac{2x^2-x-1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

Clearing the denominators, we get...

$$2x^2-x-1 = A(x+1)^2 + Bx(x+1) + Cx$$

Plugging in the roots we get...

$$\begin{aligned} x=0: &\quad 2(0^2)-0-1 = A(0+1)^2 + B(0)(0+1) + C(0) \quad \text{so that} \quad A=-1 \\ x=-1: &\quad 2(-1)^2-(-1)-1 = A(-1+1)^2 + B(-1)(-1+1) + C(-1) \quad \text{so that} \quad -C=2 \end{aligned}$$

Now we have that...

$$2x^2-x-1 = -1(x+1)^2 + Bx(x+1) - 2x \quad \text{multiplying out} \quad 2x^2-x-1 = -x^2-2x-1+Bx^2+Bx-2x$$

This means that $3x^2+3x = Bx^2+Bx$ so $B=3$.

$$\int \frac{2x^2-x-1}{x(x+1)^2}\,dx = \int \frac{-1}{x} + \frac{3}{x+1} + \frac{-2}{(x+1)^2}\,dx = \int -\frac{1}{x} + \frac{3}{x+1} - 2(x+1)^{-2}\,dx$$

Answer: $\displaystyle\int \frac{2x^2-x-1}{x(x+1)^2}\,dx = -\ln|x| + 3\ln|x+1| + 2(x+1)^{-1} + C$

5. (12 points) We know that the area inside the circle $x^2+y^2=4$ is 4π. Show this using a definite integral. At some point your work should involve a trigonometric substitution.

First, let's solve the equation for y and get $y=\pm\sqrt{4-x^2}$. The area of the circle is exactly the area between these two curves. Notice that they intersect when $\sqrt{4-x^2}=-\sqrt{4-x^2}$ so that $2\sqrt{4-x^2}=0$ so that $4-x^2=0$ thus $x=\pm 2$.

$$\text{Area} = \int_a^b \text{top} - \text{bottom} = \int_{-2}^{2} \sqrt{4-x^2} - \left(-\sqrt{4-x^2}\right)\,dx = \int_{-2}^{2} 2\sqrt{4-x^2}\,dx = 4\int_0^2 \sqrt{4-x^2}\,dx$$

Note: The last equality is due to the fact that we have are integrating an even function over a symmetric interval. Seeing the "$4-x^2$", we should immediately think of a trig substitution such as $x=2\sin(u)$ so that $dx=2\cos(u)\,du$. We must also change the bounds: $x=0$ implies $0=2\sin(u)$ so that $u=0$ and $x=2$ implies $2=2\sin(u)$ so that $\sin(u)=1$ so $u=\pi/2$.

$$\text{Area} = \cdots = 4\int_0^{\pi/2} \sqrt{4-(2\sin(u))^2}\,2\cos(u)\,du = 8\int_0^{\pi/2} \sqrt{4-4\sin^2(u)}\,\cos(u)\,du = 8\int_0^{\pi/2} \sqrt{4\cos^2(u)}\,\cos(u)\,du$$

$$= 16\int_0^{\pi/2} \cos^2(u)\,du = 8\int_0^{\pi/2} 1+\cos(2u)\,du = 8u + 4\sin(2u)\Big|_0^{\pi/2} = \left(8\frac{\pi}{2} + 4\sin\left(2\frac{\pi}{2}\right)\right) - (8(0)+4\sin(0)) = 4\pi$$

where we used the double angle identity in order to integrate $\cos^2(u)$.

Math 1120-108 **Test #1** **February 10^{th}, 2014**

Name: ANSWER KEY **Be sure to show your work!**

$\sin^2(x) = \frac{1}{2}\Big(1 - \cos(2x)\Big)$ $\frac{d}{dx}\Big[\tan(x)\Big] = \sec^2(x)$ $\frac{d}{dx}\Big[\text{asin}(x)\Big] = \frac{1}{\sqrt{1-x^2}}$

1. (27 points) Basic Integrals

(a) $\displaystyle\int x^2\cos(3x)\,dx$

The natural way to approach this integral is via *integration by parts*. Seeing the "x^2" next to the cosine indicates we'll have to do by-parts twice. First, choose $u = x^2$ and $dv = \cos(3x)\,dx$ (since we can integrate either part, we choose a u which becomes simpler when differentiated). Thus $du = 2x\,dx$ and $v = \frac{1}{3}\sin(3x)$.

$$\int x^2\cos(3x)\,dx = uv - \int v\,du = x^2\cdot\frac{1}{3}\sin(3x) - \int\frac{1}{3}\sin(3x)2x\,dx$$

Now let $u = -\frac{2}{3}x\,dx$ and $dv = \sin(3x)\,dx$ so that $du = -\frac{2}{3}$ and $v = -\frac{1}{3}\cos(3x)$.

$$\cdots = \frac{1}{3}x^2\sin(3x) - \frac{2}{3}x\left(-\frac{1}{3}\cos(3x)\right) - \int -\frac{1}{3}\cos(3x)\left(-\frac{2}{3}\right)\,dx = \frac{1}{3}x^2\sin(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + C$$

Alternatively, this problem can be tackled using an "undetermined coefficients" technique. The derivatives of $x^2\cos(3x)$ fall into the pattern $(Ax^2+Bx+C)\cos(3x) + (Dx^2+Ex+F)\sin(3x)$ so we guess that the integral has an answer of this form. To finish this way we would need to differentiate our guess, match it with the integrand, and then solve for A through F. However, this isn't very practical in this case, so we'll stick to the standard technique.

Answer: $\displaystyle\int x^2\cos(3x)\,dx = \frac{1}{3}x^2\sin(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + C$

(b) $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

This is just a u-substitution problem. Notice that the derivative of $\sqrt{x} = x^{1/2}$ is $\frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$. So a multiple of the derivative of the exponent appears in the integrand. Thus we should let $u = \sqrt{x}$ so that $du = \frac{1}{2\sqrt{x}}\,dx$ so that $2\,du = \frac{dx}{\sqrt{x}}$. Since this is a definite integral, we should also change the bounds: $x = 1 \mapsto u = \sqrt{1} = 1$ and $x = 4 \mapsto u = \sqrt{4} = 2$.

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int_1^2 e^u\cdot 2\,du = 2e^u\Big|_1^2 = \boxed{2e^2 - 2e}$$

(c) $\displaystyle\int_0^\pi \cos^2(x)\,dx$

This is an even power of cosine, so we'll need to use a half-angle identity to be able to integrate it. Recall that $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$.

$$\int_0^\pi \cos^2(x)\,dx = \int_0^\pi \frac{1}{2} + \frac{1}{2}\cos(2x)\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x)\Big|_0^\pi = \left(\frac{\pi}{2} + \frac{1}{4}\sin(2\pi)\right) - \left(\frac{0}{2} + \frac{1}{4}\sin(0)\right) = \boxed{\frac{\pi}{2}}$$

2. (30 points) More Integrals

(a) $\displaystyle\int \arcsin(x)\,dx$

Integration by parts will work here. Since we are trying to integrate $\arcsin(x)$, this needs to be our u. Thus we let $u = \arcsin(x)$ and $dv = dx$ so that $du = \frac{1}{\sqrt{1-x^2}}\,dx$ and $v = x$.

$$\int \arcsin(x)\,dx = uv - \int v\,du = \arcsin(x)\cdot x - \int x\frac{1}{\sqrt{1-x^2}}\,dx$$

Next, to finish this integral we need to use a simple substitution: $u = 1-x^2$ so that $du = -2x\,dx$ and so $\frac{1}{2}\,du = -x\,dx$.

$$-\int x\frac{1}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{u}}\cdot\frac{1}{2}\,du = \int \frac{1}{2}u^{-1/2}\,du = u^{1/2} + C = \sqrt{1-x^2} + C$$

Answer: $\displaystyle\int \arcsin(x)\,dx = x\arcsin(x) + \sqrt{1-x^2} + C$

(b) $\displaystyle\int \sin^6(x)\cos^3(x)\,dx$

We have an even power of sine and an odd power of cosine. We can turn an even number of powers of cosine into sines using the identity $\cos^2(x) = 1 - \sin^2(x)$. This leaves 1 cosine to serve as the du in our substitution. We'll use $u = \sin(x)$ and $du = \cos(x)\,dx$.

$$\int \sin^6(x)\cos^3(x)\,dx = \int \sin^6(x)\cos^2(x)\cos(x)\,dx = \int \sin^6(x)(1-\sin^2(x))\cos(x)\,dx = \int u^6(1-u^2)\,du$$

$$= \int (u^6 - u^8)\,du = \frac{1}{7}u^7 - \frac{1}{9}u^9 + C = \boxed{\frac{1}{7}\sin^7(x) - \frac{1}{9}\sin^9(x) + C}$$

(c) $\displaystyle\int e^{-x}\cos(x)\,dx$

This integral can be tackled by using integration by parts twice and then solving for the integral. Let $u = \cos(x)$ and $dv = e^{-x}\,dx$ so that $du = -\sin(x)\,dx$ and $v = -e^{-x}$.

$$\int e^{-x}\cos(x)\,dx = uv - \int v\,du = \cos(x)(-e^{-x}) - \int -e^{-x}(-\sin(x))\,dx$$

Next, let $u = \sin(x)$ and $dv = -e^{-x}\,dx$ so that $du = \cos(x)\,dx$ and $v = e^{-x}$.

$$I = \cdots = -e^{-x}\cos(x) + \sin(x)e^{-x} - \int e^{-x}\cos(x)\,dx = -e^{-x}\cos(x) + e^{-x}\sin(x) - I$$

Thus $2I = -e^{-x}\cos(x) + e^{-x}\sin(x)$.

Answer: $\displaystyle\int e^{-x}\cos(x)\,dx = -\frac{1}{2}e^{-x}\cos(x) + \frac{1}{2}e^{-x}\sin(x) + C$

Alternatively we could use an "undetermined coefficients" technique as outlined in the answer key for section 101.

3. (8 points) Write down the "forms" we would use to find the partial fraction decomposition of...

$$\frac{-3x^5+2x^4-x^2+7}{x^2(x-3)(x^2+x+5)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3} + \frac{Dx+E}{x^2+x+5} + \frac{Fx+G}{(x^2+x+5)^2}$$

4. (28 points) More Integrals

(a) $\displaystyle\int \frac{1}{x^2+6x+13}\,dx$

A u-substitution will not work here (a multiple of the derivative of the denominator does not appear as the numerator), so we should consider partial fractions. However, $x^2+6x+13$ does not factor (it has complex roots since $B^2-4AC = 6^2-4(1)(13) = -16 < 0$). Thus this must be the derivative of arc tangent in disguise. We should complete the square.

$$\int \frac{1}{x^2+6x+13}\,dx = \int \frac{1}{(x+3)^2+4}\,dx = \int \frac{1}{4}\cdot\frac{1}{\frac{(x+3)^2}{4}+1}\,dx = \int \frac{1}{4}\cdot\frac{1}{\left(\frac{x+3}{2}\right)^2+1}\,dx$$

Next, substitute $u = \frac{x+3}{2}$ and so $du = \frac{1}{2}\,dx$ thus $2\,du = dx$.

$$\int \frac{1}{x^2+6x+13}\,dx = \cdots = \int \frac{1}{4}\cdot\frac{1}{u^2+1}\cdot 2\,du = \frac{1}{2}\text{arctan}(u)+C = \boxed{\frac{1}{2}\arctan\left(\frac{x+3}{2}\right)+C}$$

(b) $\displaystyle\int \frac{2x^2+3x-7}{(x-3)(x^2+1)}\,dx$

This is an obvious candidate for partial fractions.

$$\frac{2x^2+3x-7}{(x-3)(x^2+1)} = \frac{A}{x-3}+\frac{Bx+C}{x^2+1}$$

Clearing the denominators, we get...

$$2x^2+3x-7 = A(x^2+1)+(Bx+C)(x-3)$$

Plugging in $x=3$, we get $2(3^2)+3(3)-7 = A(3^2+1)+(B(3)+C)(3-3)$ so that $20 = 10A$ so that $A=2$. This means that

$$2x^2+3x-7 = 2(x^2+1)+(Bx+C)(x-3) \quad \text{so that} 2x^2+3x-7 = 2x^2+2+Bx^2-3Bx+Cx-3C$$

Thus $0x^2+3x-9 = Bx^2+(-3B+C)x-3C$ thus $B=0$, $-3B+C=3$, and $-3C=-9$, so $C=3$.

$$\int \frac{2x^2+3x-7}{(x-3)(x^2+1)}\,dx = \int \frac{2}{x-3}+\frac{3}{x^2+1}\,dx = \boxed{2\ln|x-3|+3\arctan(x)+C}$$

(c) $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

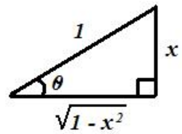
Solution 1: Notice the $\sqrt{1-x^2}$. This yells out, "Trig. substitution!" Let $x=\sin(u)$ and so $dx=\cos(u)\,du$. Notice $\sqrt{1-x^2} = \sqrt{1-\sin^2(u)} = \sqrt{\cos^2(u)} = \cos(u)$.

$$\int \frac{x^3}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^3(u)}{\cos(u)}\cos(u)\,du = \int \sin^3(u)\,du$$

Now we have an odd power of sine, so we use the substitution $w=\cos(u)$ (so that $dw=-\sin(u)\,du$).

$$\int \sin^3(u)\,du = \int (1-\cos^2(u))\,\sin(u)\,du = \int (1-w^2)(-dw) = \frac{1}{3}w^3-w+C$$

Substitute back and get $(1/3)\cos^3(u)-\cos(u)+C$ and $u=\arcsin(x)$ so we get $(1/3)\cos^3(\arcsin(x))-\cos(\arcsin(x))+C$ (which is an acceptible answer). But let's clean up this answer a little more.



For this triangle $\sin(\theta) = x/1 = x$ so $\arcsin(x) = \theta$. Notice that $\cos(\arcsin(x)) = \cos(\theta) = \sqrt{1-x^2}/1 = \sqrt{1-x^2}$

Answer: $\frac{1}{3}(1-x^2)^{3/2} - \sqrt{1-x^2} + C$

Solution 2: Again, get rid of $\sqrt{1-x^2}$, but this time use $u^2 = 1-x^2$ so that $2u\,du = -2x\,dx$ and thus $-u\,du = x\,dx$.

$$\int \frac{x^2}{\sqrt{1-x^2}} x\,dx = \int \frac{1-u^2}{\sqrt{u^2}}(-u\,du) = \int \frac{u^3-u}{u}\,du = \int u^2 - 1\,du = (1/3)u^3 - u + C$$

Finally, recall that $u = \sqrt{1-x^2}$ and substitute back and get $(1/3)(1-x^2)^{3/2} - \sqrt{1-x^2} + C$

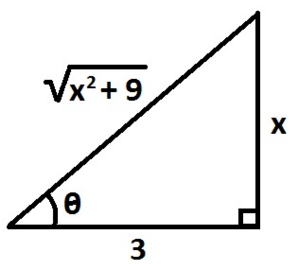
Solution 3: Use integration by parts, $u = x^2$ and $dv = \frac{x}{\sqrt{1-x^2}}\,dx$ so that $du = 2x\,dx$ and $v = -\sqrt{1-x^2}$.

Note: To integrate $x/\sqrt{1-x^2}$ and $2x\sqrt{1-x^2}$ you can use the substitution $w = 1-x^2$.

$$\int \frac{x^2}{\sqrt{1-x^2}} x\,dx = x^2(-\sqrt{1-x^2}) - \int -\sqrt{1-x^2}2x\,dx = -x^2\sqrt{1-x^2} + \int 2x\sqrt{1-x^2}\,dx$$

$= -x^2\sqrt{1-x^2} - (2/3)(1-x^2)^{3/2} + C = -\sqrt{1-x^2} + \sqrt{1-x^2} - x^2\sqrt{1-x^2} - (2/3)(1-x^2)^{3/2} + C = -\sqrt{1-x^2} + (1-x^2)\sqrt{1-x^2} - (2/3)(1-x^2)^{3/2} + C = -\sqrt{1-x^2} + (1-x^2)^{3/2} - (2/3)(1-x^2)^{3/2} + C = -\sqrt{1-x^2} + (1/3)(1-x^2)^{3/2} + C$ (again the same answer).

5. (7 points) Simplify $\sin\left(\arctan\left(\frac{x}{3}\right)\right)$. Draw a triangle to back up your answer.



We would like the tangent of some angle to be $\frac{x}{3}$, so we draw a triangle whose opposite side has length x and adjacent side has length 3. By the Pythagorean theorem, the hypotenuse must be $\sqrt{x^2+3^2} = \sqrt{x^2+9}$. Finally, recall that sine is just opposite over hypotenuse.

Answer: $\sin\left(\arctan\left(\frac{x}{3}\right)\right) = \sin(\theta) = \boxed{\frac{x}{\sqrt{x^2+9}}}$