

Name: _____

Be sure to show your work!

1. (14 points) Approximating integrals

- (a) Consider $I = \int_{-3}^6 \sin(\ln(x^2 + 1)) dx$. Write down the midpoint approximation for I if $n = 3$ (i.e. M_3).
[Don't worry about simplifying.]

- (b) Let $I = \int_0^{\sqrt{2}} e^{-x^2/4} dx$. (The graph of $y = e^{-x^2/4}$ is decreasing and concave down on $[0, \sqrt{2}]$.)

Rank L_n , R_n , M_n , T_n , and I from smallest to largest (with “ \leq ” signs in between).For example: $L_n \leq R_n \leq M_n \leq T_n \leq I$ (which is definitely *not* the right answer).

$$\boxed{} \leq \boxed{} \leq \boxed{} \leq \boxed{} \leq \boxed{}$$

2. (14 points) Improper integrals

- (a) Does the following integral converge or diverge? If it converges, find its value.

$$\int_0^{\infty} x e^{-x^2} dx$$

- (b) Does the following integral converge or diverge? SHOW YOUR WORK! [Don't worry about its value.]

$$\int_0^2 \frac{2 + \cos(t)}{t^2} dt$$

3. (12 points) Consider a pyramid with height 5 and square base which is 10 by 10.

(a) Slice the pyramid horizontally. Let x be the distance from the ground. Find a formula for the volume of a slice. Draw some picture to back up your work.

(b) Using part (a), write down an integral which computes the volume of the pyramid.
[Don't worry about evaluating this integral.]

4. (14 points) Consider the region bounded by $y = x^2$ and $y = 2x$. Write down an integral which computes the volume of the solid obtained by rotating this region about...

(a) ... the axis $y = -3$. [Don't worry about evaluating this integral.]

(b) ... the y -axis (i.e. $x = 0$). [Don't worry about evaluating this integral.]

5. (10 points) Set up the integral which computes the arc length of $y = \ln(x)$ when $1 \leq x \leq 15$.
[Don't worry about evaluating this integral.]

6. (12 points) A rod with density $\delta(x) = 1 + x$ lies on the x -axis between $x = 0$ and $x = 2$. Find the center of mass of the rod.

7. (12 points) A cylindrical barrel is partially filled with ooze (which has density 100 lbs. per cubic foot). The barrel is 4 feet tall, has a radius of 2 feet, and is 75% filled (the ooze is 3 feet deep). Find the work required to pump the ooze up over the top edge of the barrel. Sketch a picture which indicates how you went about computing work.

8. (12 points) Let $p(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$.

(a) Show that $p(x)$ is a probability density function (pdf).

(b) Find the cumulative distribution function and the median of this pdf.

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1. (14 points) Approximating integrals

- (a) Consider $I = \int_{-6}^3 \sqrt{e^x + 1} dx$. Write down the trapezoid rule approximation for I if $n = 3$ (i.e. T_3).
[Don't worry about simplifying.]

- (b) Let $I = \int_{-\sqrt{2}}^0 e^{-x^2/4} dx$. (The graph of $y = e^{-x^2/4}$ is increasing and concave down on $[-\sqrt{2}, 0]$.)

Rank L_n , R_n , M_n , T_n , and I from smallest to largest (with “ \leq ” signs in between).For example: $L_n \leq R_n \leq M_n \leq T_n \leq I$ (which is definitely *not* the right answer).

$$\boxed{} \leq \boxed{} \leq \boxed{} \leq \boxed{} \leq \boxed{}$$

2. (14 points) Improper integrals

- (a) Does the following integral converge or diverge? If it converges, find its value.

$$\int_0^4 \frac{1}{(x-2)^2} dx$$

- (b) Does the following integral converge or diverge? SHOW YOUR WORK! [Don't worry about its value.]

$$\int_2^\infty \frac{2 + \cos(t)}{t^2} dt$$

3. (12 points) Consider a pyramid with height 3 and square base which is 5 by 5.

(a) Slice the pyramid horizontally. Let x be the distance from the ground. Find a formula for the volume of a slice. Draw some picture to back up your work.

(b) Using part (a), write down an integral which computes the volume of the pyramid.
[Don't worry about evaluating this integral.]

4. (14 points) Consider the region bounded by $y = x^2$ and $y = 3x$. Write down an integral which computes the volume of the solid obtained by rotating this region about...

(a) ... the axis $y = -1$. [Don't worry about evaluating this integral.]

(b) ... the y -axis (i.e. $x = 0$). [Don't worry about evaluating this integral.]

5. (10 points) Set up the integral which computes the arc length of $y = \sin(x)$ when $0 \leq x \leq 2\pi$.
[Don't worry about evaluating this integral.]

6. (12 points) A rod with density $\delta(x) = 2x$ lies on the x -axis between $x = 0$ and $x = 3$. Find the center of mass of the rod.

7. (12 points) Bob is pulling some paint buckets up to his roof using some chain. Each bucket weighs 5 lbs. and the chain weighs 2 lbs. per foot. If Bob's roof is 20 feet above the ground and the chain is 20 feet long, how much work is required to lift the bucket and chain?

8. (12 points) Let $p(x) = \begin{cases} 5e^{-5x} & x \geq 0 \\ 0 & x < 0 \end{cases}$.

(a) Show that $p(x)$ is a probability density function (pdf).

(b) Find the mean of this pdf.