Math 2110-101 | # Test #1 | **September 16^{th}, 2020**

Name: ______________________________ **Be sure to show your work!**

1. (25 points) "Fun" with Truth Tables!

(a) Fill out an abbreviated truth table for the following statement: $(a \vee \neg a) \to (b \wedge \neg b)$. Circle your concluding truth values and the correct answer:

This statement is a **Contingency** / **Contradiction** / **Tautology**.

(b) Show that $\neg a \vee \neg b$ and $\neg(a \wedge b)$ are logically equivalent.

(c) Consider the following statement: $\neg a, a \to b \vdash \neg b$. Circle the correct answers.

IS / **IS NOT** a theorem of L by the **Soundness** / **Completeness** theorem.

2. (25 points) System L

(a) I have provided a proof of Lemma L6 below – except the justifications are missing.

Fill in the justifications and please **be specific!**

Lemma L6 $A \to (B \to C), B \vdash_L A \to C$

1: $A \to (B \to C)$ ____________________

2: B ____________________

3: $A \to B$ ____________________

4: $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$ ____________________

5: $(A \to B) \to (A \to C)$ ____________________

6: $A \to C$ ____________________

(b) Prove Lemma L10: $\vdash_L (\neg B \to \neg A) \to (A \to B)$. [You may use the deduction theorem if you wish.]

Hint/Suggestion: Axiom 3 should be helpful.

3. (25 points) Models, Variables, Free for...

(a) Consider the formula: $\forall x\,(A(f(\underline{a}, x), z) \to \exists y\,(B(y) \land C(x, y, z)))$.

i. Underline the scope of each quantifier in the above formula.

ii. **Circle** all of the **bound variables** in the above formula.

iii. Circle the correct answers and fill in the blanks below:

The formula above **IS** / **IS NOT** a sentence since ____________________________________.

The term $t = g(y, \underline{a})$ **IS** / **IS NOT** free for z in the formula above.

(b) Consider the formula: $(\forall x\, A(x, \underline{c})) \to (\exists y\, B(y))$.

Translate the above formula into *plain English* when our model consists of the universe of all people, $A(x, y) :=$ "x loves y", $B(x) :=$ "x likes the Middle", and $\underline{c} :=$ "Raymond".

(c) Consider the formula: $(\forall x\, A(x)) \lor (\exists z\, B(f(z), \underline{c}))$.

My model uses the universe of natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$ and defines $B(x, y) :=$ "$x < y$". Finish specifying my model in such a way that we prove the above statement is not logically valid.

4. (25 points) System K

(a) I have provided a proof of Lemma K17 below – except the justifications are missing.

Fill in the justifications.

Lemma K17 $\forall x \neg A(x) \vdash_K \neg \exists x\, A(x)$

1: $\forall x \neg A(x)$ ________________________

2: $\forall x \neg A(x) \to \neg\neg\forall x \neg A(x)$ ________________________

3: $\neg\neg\forall x \neg A(x)$ ________________________

4: $\neg \exists x\, A(x)$ ________________________

(b) Prove Lemma K22: $\vdash_K \exists x \forall y\, A(x, y) \to \forall y \exists x\, A(x, y)$.

Math 2110-102 # Test #1 **September 17^{th}, 2020**

Name: ________________________________ **Be sure to show your work!**

1. (25 points) "Fun" with Truth Tables!

(a) Fill out an abbreviated truth table for the following statement: $(a \vee \neg b) \rightarrow (b \wedge \neg a)$. Circle your concluding truth values and the correct answer:

This statement is a **Contingency** / **Contradiction** / **Tautology**.

(b) Show that $\neg a \wedge \neg b$ and $\neg(a \vee b)$ are logically equivalent.

(c) Consider the following statement: $\neg b, a \rightarrow b \vdash \neg a$. Circle the correct answers.

IS / **IS NOT** a theorem of L by the **Soundness** / **Completeness** theorem.

2. (25 points) System L

(a) I have provided a proof of Lemma L8 below – except the justifications are missing.

Fill in the justifications and please **be specific!**

Lemma L8 $A \to B, B \to C \vdash_L A \to C$

1: $A \to B$ ____________________

2: $B \to C$ ____________________

3: $A \to (B \to C)$ ____________________

4: $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$ ____________________

5: $(A \to B) \to (A \to C)$ ____________________

6: $A \to C$ ____________________

(b) Prove Lemma L10: $\vdash_L (\neg B \to \neg A) \to (A \to B)$. [You may use the deduction theorem if you wish.]

Hint/Suggestion: Axiom 3 should be helpful.

3. (25 points) Models, Variables, Free for...

(a) Consider the formula: $\exists x\,(A(f(z), x) \rightarrow \forall y\,(B(x, y) \wedge C(\underline{a}, y, w)))$.

i. Underline the scope of each quantifier in the above formula.

ii. **Circle** all of the **bound variables** in the above formula.

iii. Circle the correct answers and fill in the blanks below:

The formula above **IS** / **IS NOT** a sentence since ____________________________________.

The term $t = g(y, \underline{a})$ **IS** / **IS NOT** free for z in the formula above.

(b) Consider the formula: $(\exists x\, A(x, \underline{c})) \rightarrow (\forall y\, B(y))$.

Translate the above formula into *plain English* when our model consists of the universe of all people, $A(x, y) :=$ "x rescues y", $B(x) :=$ "x will cheer", and $\underline{c} :=$ "Princess Fiona".

(c) Consider the formula: $(\forall x\, A(x, \underline{c})) \vee (\exists z\, B(f(z)))$.

My model uses the universe of natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$ and defines $A(x, y) :=$ "$x \geq y$". Finish specifying my model in such a way that we prove the above statement is not logically valid.

4. (25 points) System K

(a) I have provided a proof of Lemma K17 below – except the justifications are missing.

Fill in the justifications.

Lemma K17 $\forall x\, \neg A(x) \vdash_K \neg\exists x\, A(x)$

1: $\forall x\, \neg A(x)$ ______________________________

2: $\forall x\, \neg A(x) \to \neg\neg\forall x\, \neg A(x)$ ______________________________

3: $\neg\neg\forall x\, \neg A(x)$ ______________________________

4: $\neg\exists x\, A(x)$ ______________________________

(b) Prove Lemma K22: $\vdash_K \exists x\, \forall y\, A(x,y) \to \forall y\, \exists x\, A(x,y)$.