Math 2130-101 **Final Exam for Section 101** **December 11^{th}, 2009**

Name: ______________________________ **Be sure to show your work!**

1. (____/10 points) Let ℓ_1 be the line parameterized by $\mathbf{r}_1(t) = (t+2, -6t-5, 4t)$ and let ℓ_2 be the line parameterized by $\mathbf{r}_2(t) = (t, -3t+7, 2t-8)$. Then ℓ_1 and ℓ_2 are ...

parallel / intersecting / skew / the same line(s).

Circle the correct answer. [Note: If you don't show any work, you will not get any credit.]

2. (_____/10 points) Let $f(x, y, z) = x - y + z^2$. Find the maximum and minimum **values** of f if f's inputs are constrained to $x^2 + y^2 + z^2 = 2$.

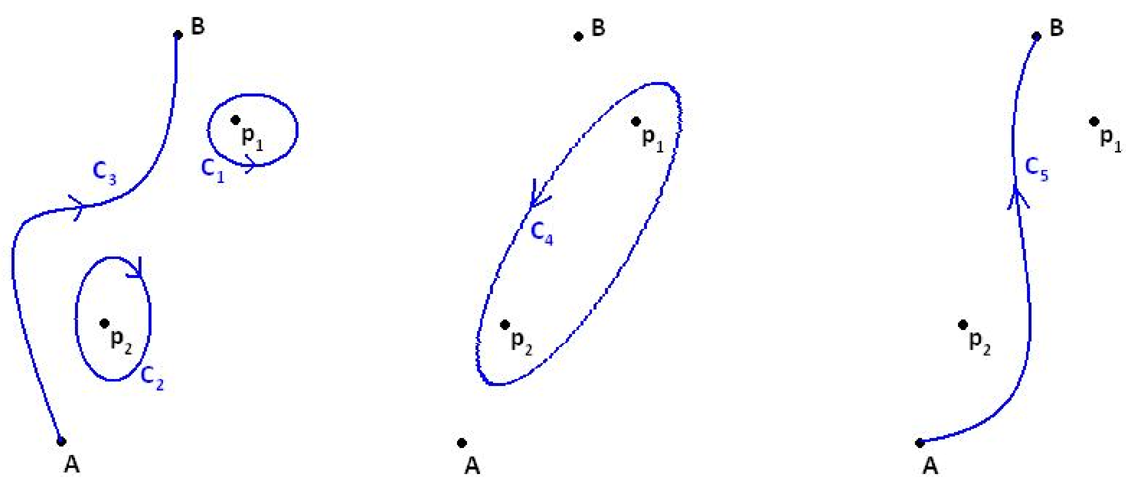
3. (____/10 points) Consider the function $f(x,y) = \dfrac{e^{-(x^2+y^2)}}{\pi}$

It's easy to see that $f(x,y) \geq 0$ everywhere. Compute $\displaystyle\iint_{\mathbb{R}^2} f(x,y)\, dA$ and decide if f is a probability distribution function.

Is f a probability distribution function? YES / NO

4. (_____/8 points) Let C be the circle $(x-1)^2 + y^2 = 4$. Evaluate the integral $\int_C y^2\, ds$

5. (_____/8 points) Suppose that $\mathbf{F}(x,y) = (P(x,y), Q(x,y))$ is a vector field such that $P_y = Q_x$ except at the points p_1 and p_2. Let $C_1, \ldots, C_5$ be the curves described in the picture below. Also, suppose that we know $\int_{C_1} \mathbf{F} \cdot d\mathbf{X} = 2$, $\int_{C_2} \mathbf{F} \cdot d\mathbf{X} = 5$, and $\int_{C_3} \mathbf{F} \cdot d\mathbf{X} = 1$.



(a) $\int_{C_4} \mathbf{F} \cdot d\mathbf{X} =$ ________________

(b) $\int_{C_5} \mathbf{F} \cdot d\mathbf{X} =$ ________________

6. (____/10 points) For each of the following vector fields, decide if **F** is conservative. If **F** is conservative, find a potential function.

(a) $\mathbf{F}(x, y) = (2xy + 2xe^{x^2}, x^2 + 3y^2)$

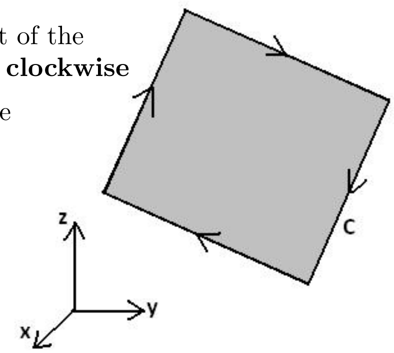
(b) $\mathbf{F}(x, y, z) = (yz + e^y + y, xz + xe^y + 3y^2, xy)$

7. (_____/10 points) Let C be the circle $x^2 + y^2 = 1$ oriented counter-clockwise.

Evaluate $\displaystyle\int_C (e^{\sqrt{x}} - y^3)\,dx + (x^3 + \sqrt{y^3 + 7} + \arctan(y^2 + 5))\,dy$

8. (____/12 points) Find the centroid of the upper-half of the unit sphere $x^2 + y^2 + z^2 = 1$.

9. (____/12 points) Let C be the rectangular boundary of the part of the plane $x + 2y + z = 1$ where $0 \le x \le 2$ and $0 \le y \le 1$. C is oriented **clockwise** when viewed from above. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{X}$ where $\mathbf{F}(x, y, z) = (y + \sqrt{x^2+1}, z + x^2, 2y + e^{-z^2})$.



10. (____/10 points) Let S_1 be the sphere $x^2 + y^2 + z^2 = 4$ oriented outward. Evaluate the flux integral $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = (y^2 - \sqrt{z^2+1}, e^{x+z}, 3z + \sqrt{x^3 + 10y^2})$.

Math 2130-102 **Final Exam for** Section 102 **December** 15^{th}**, 2009**

Name: ______________________________ **Be sure to show your work!**

1. (____/10 points) Let ℓ_1 be the line parameterized by $\mathbf{r}_1(t) = (2t+2, -2t+3, 4t+4)$ and let ℓ_2 be the line parameterized by $\mathbf{r}_2(t) = (-t, t+1, -2t-1)$. Then ℓ_1 and ℓ_2 are ...

parallel / intersecting / skew / the same line(s).

Circle the correct answer. [Note: If you don't show any work, you will not get any credit.]

2. (____/10 points) Let $f(x, y) = 2x^2 + y^2 - 2x$.

(a) Find and classify all of the critical (i.e. stationary) points of f.
[Determine if each point is a relative minimum, relative maximum, or a saddle point.]

(b) Find the absolute maximum and minimum **value** of f on the disk $x^2 + y^2 \leq 16$.
[Use Lagrange Multipliers to determine what happens on the boundary of the disk.]

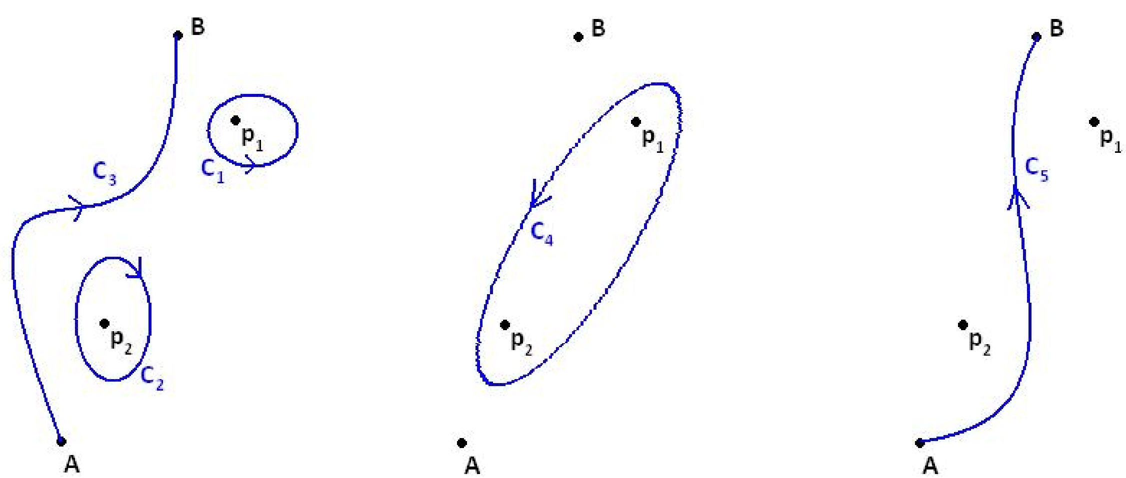
3. (____/10 points) Consider the function $f(x,y) = e^{-(x^2+y^2)}$

It's easy to see that $f(x,y) \geq 0$ everywhere. Compute $\iint_{\mathbb{R}^2} f(x,y)\, dA$ and decide if f is a probability distribution function.

Is f a probability distribution function? YES / NO

4. (____/8 points) Let C be the curve parameterized by $\mathbf{X}(t) = (2t, 3, t^2)$ where $0 \le t \le 1$. Evaluate the integral $\int_C xy\, ds$

5. (____/8 points) Suppose that $\mathbf{F}(x, y) = (P(x, y), Q(x, y))$ is a vector field such that $P_y = Q_x$ except at the points p_1 and p_2. Let $C_1, \ldots, C_5$ be the curves described in the picture below. Also, suppose that we know $\int_{C_1} \mathbf{F} \cdot d\mathbf{X} = 3$, $\int_{C_2} \mathbf{F} \cdot d\mathbf{X} = -2$, and $\int_{C_3} \mathbf{F} \cdot d\mathbf{X} = 5$.



(a) $\int_{C_4} \mathbf{F} \cdot d\mathbf{X} =$ ______________

(b) $\int_{C_5} \mathbf{F} \cdot d\mathbf{X} =$ ______________

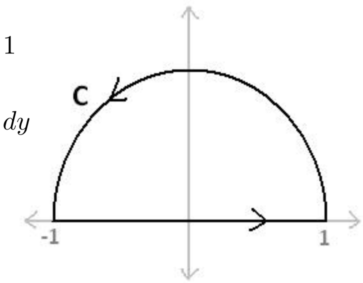
6. (____/10 points) For each of the following vector fields, decide if $\mathbf{F}$ is conservative. If $\mathbf{F}$ is conservative, find a potential function.

(a) $\mathbf{F}(x, y) = (2xe^y + 3x^2, x^2e^y + (1+y)e^y + x)$

(b) $\mathbf{F}(x, y, z) = (yz + 2x, xz + e^z, xy + ye^z)$

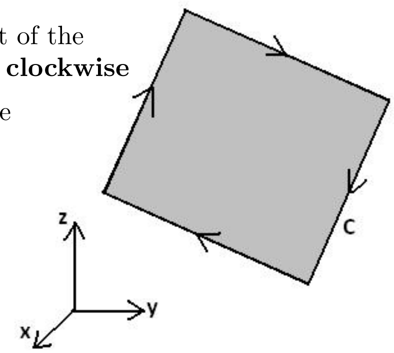
7. (____/10 points) Let C be the upper-half of the circle $x^2 + y^2 = 1$ along with the x-axis from -1 to 1 oriented counter-clockwise.

Evaluate $\displaystyle\int_C (-2y + \arctan(x^2+2) + \sqrt{x^3 + \sin(x)})\,dx + (x^2 + e^{-y^2})\,dy$



8. (____/12 points) Find the centroid of S_1 where S_1 is the part of the cone $z = \sqrt{x^2 + y^2}$ which lies below the plane $z = 4$. S_1 is a **surface** with surface area $16\pi\sqrt{2}$.

9. (____/12 points) Let C be the rectangular boundary of the part of the plane $2x + y + z = 1$ where $0 \leq x \leq 1$ and $0 \leq y \leq 2$. C is oriented **clockwise** when viewed from above. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{X}$ where $\mathbf{F}(x, y, z) = (2y - z^2 + e^{\sin(x)+1}, \sqrt{y^6 + 1}, y + \tan(\sqrt{z^2 + 1}))$.



10. (_____/10 points) Let S_1 be the upper hemi-sphere $x^2 + y^2 + z^2 = 1$ oriented upward. Also, let S_2 be the unit disk in the xy-plane ($z = 0$ and $x^2 + y^2 \leq 1$) oriented upward as well. Suppose that $\iint_{S_2} \mathbf{F} \cdot d\mathbf{S} = \frac{3}{5}\pi$ where $\mathbf{F}$ is a vector field whose divergence is $\text{div}(\mathbf{F}) = x^2 + y^2 + z^2$.

Evaluate the flux integral $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S}$.