Math 2130-101 # Test #1 for Section 101 **September 21^{st}, 2009**

Name: ______________________________________ **Be sure to show your work!**

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} \qquad\qquad \text{Arc Length} = \int_a^b |\mathbf{r}'(t)|\,dt$$

1. (____/22 points) Let $\mathbf{u} = (1, -1, 1)$ and $\mathbf{v} = (-1, 2, 4)$.

(a) Compute the following:

i. $|\mathbf{u}| =$

ii. $|\mathbf{v}| =$

iii. $\mathbf{u}\cdot\mathbf{v} =$

iv. $\mathbf{u}\times\mathbf{v} =$

v. $\text{proj}_{\mathbf{v}}(\mathbf{u}) =$

(b) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ (don't worry about evaluating inverse trigonometric functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(c) Compute the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.

(d) Find a unit vector perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.

(e) Are $\mathbf{u}$ and $\mathbf{v}$ parallel vectors?

2. (____/17 points) Consider the two points $P = (-1, 1, 2)$ and $Q = (1, 2, 2)$ in $\mathbb{R}^3$.

(a) Find the distance between P and Q.

(b) Parameterize the line segment beginning at P and ending at Q. Make sure you include a range for t (i.e. $a \leq t \leq b$) in your parameterization.

(c) Set up and evaluate an integral which computes the arc length of this line segment.

(d) Let ℓ_1 be the line through P and Q. Let ℓ_2 be the line defined by $\mathbf{L}_2(t) = (-3 + t, t, 2 + t)$. Determine if ℓ_1 and ℓ_2 are the same, parallel, intersecting, or skew.

3. (____/16 points) Let C be the circle centered at $(0, 3)$ with radius 3.

(a) Find a (regular rectangular coordinate) scalar equation for C.

(b) Convert your equation from part (a) to a polar equation.

(c) Parameterize the upper half of this circle (oriented counter-clockwise).

4. (_____/16 points) Commander Keen just landed on a small moon located near planet Vorticon. Acceleration due to **gravity** on this small moon just happens to be $\mathbf{2\ m/s^2}$. For some reason, Keen seems to be compelled to throw a ball from the top of his 25 meter tall spaceship – so $\mathbf{p}(0) = (0, 25)$. When the ball leaves Keen's hand, it is traveling horizontally at a rate of 10 meters per second – so $\mathbf{v}(0) = (10, 0)$. Assume there is no friction (this moon has no atmosphere) and the ball is in free fall.

(a) Find the vector valued function, $\mathbf{v}(t)$, which describes the ball's velocity t seconds after it is thrown.

(b) Find the vector valued function, $\mathbf{p}(t)$, which describes the ball's position t seconds after it is thrown.

(c) When will the ball hit the ground?

5. (_____/14 points) Let Π be the plane containing the points $P = (1, 2, 1)$, $Q = (2, 2, 2)$, and $R = (0, 3, 2)$.

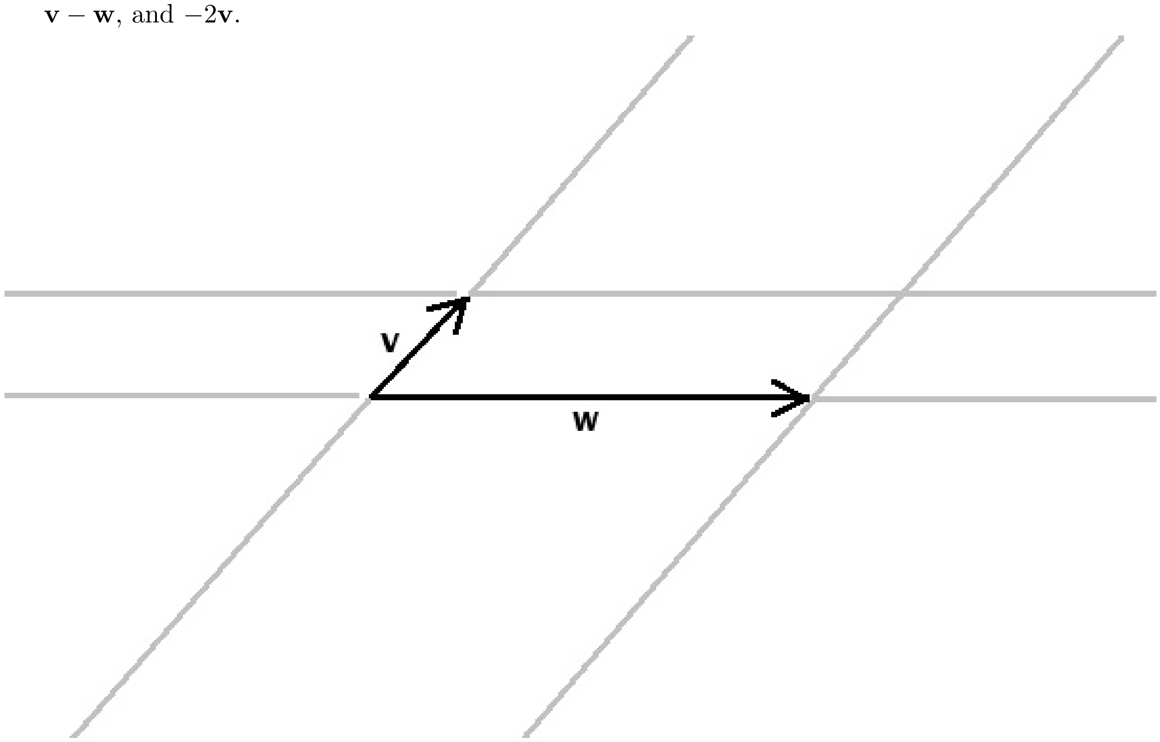
(a) Find a parameterization for the plane Π.

(b) Find a scalar equation for the plane Π.

6. (____/15 points) No numbers here.

(a) Let $\mathbf{u}$ and $\mathbf{v}$ be unit vectors. Show that $(\mathbf{u} \cdot \mathbf{v})^2 + |\mathbf{u} \times \mathbf{v}|^2 = 1$.

(b) Two vectors $\mathbf{v}$ and $\mathbf{w}$ are shown in the picture below. Sketch (with labels) the vectors $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$, and $-2\mathbf{v}$.



Math 2130-102 **Test #1 for Section 102** **September 21^{st}, 2009**

Name: ______________________________ **Be sure to show your work!**

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} \qquad \text{Arc Length } = \int_a^b |\mathbf{r}'(t)|\,dt$$

1. (____/22 points) Let $\mathbf{u} = (2, -1, 1)$ and $\mathbf{v} = (-1, 1, 3)$.

(a) Compute the following:

i. $|\mathbf{u}| =$

ii. $|\mathbf{v}| =$

iii. $\mathbf{u}\cdot\mathbf{v} =$

iv. $\mathbf{u}\times\mathbf{v} =$

v. $\text{proj}_{\mathbf{v}}(\mathbf{u}) =$

(b) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ (don't worry about evaluating inverse trigonometric functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(c) Compute the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.

(d) Find a unit vector perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.

(e) Are $\mathbf{u}$ and $\mathbf{v}$ parallel vectors?

2. (____/12 points) Consider the vector valued function defined by $\mathbf{r}(t) = (4\cos(t), 4\sin(t), 3t)$ where $-\pi \le t \le \pi$.

(a) Set up and evaluate an integral which computes this curve's arc length.

(b) Find an equation for the line tangent to $\mathbf{r}(t)$ at $t = 0$.

3. (____/8 points) Let ℓ_1 be the line parameterized by $\mathbf{L}_1(t) = (1 + t, 1 + 2t, 1)$ and ℓ_2 the line parameterized by $\mathbf{L}_2(t) = (-t, 5 + t, -1 - 2t)$. Are these lines the same, parallel, intersecting, or skew?

4. (____/7 points) Convert the polar equation $r = 5\sec(\theta)$ to a rectangular equation.

5. (____/6 points) Consider the circle $(x-1)^2 + (y+3)^2 = 4^2$. Parameterize the upper-half of this circle.

6. (____/12 points) Commander Keen just landed on a small moon located near planet Vorticon. Acceleration due to gravity on this small moon just happens to be 2 m/s^2. For some reason, Keen seems to be compelled to throw a ball from the top of his 10 meter tall spaceship – so $\mathbf{p}(0) = (0, 10)$. When the ball leaves Keen's hand, it is traveling horizontally at a rate of 5 meters per second – so $\mathbf{v}(0) = (5, 0)$. Assume there is no friction (this moon has no atmosphere) and the ball is in free fall.

(a) Find the vector valued function, $\mathbf{v}(t)$, which describes the ball's velocity t seconds after it is thrown.

(b) Find the vector valued function, $\mathbf{p}(t)$, which describes the ball's position t seconds after it is thrown.

7. (____/18 points) Let Π be the plane containing the points $P = (1, 2, 1)$, $Q = (3, 3, 2)$, and $R = (1, -1, 4)$.

(a) Find a parameterization for Π.

(b) Find a scalar equation for Π.

(c) Consider the plane Π_2 defined by $x - y - z = 0$. Are the planes Π and Π_2 parallel, perpendicular, or neither?

8. (____/15 points) Let $\mathbf{u}$ and $\mathbf{v}$ be vectors of **equal length**. Show $(\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}-\mathbf{v}) = 0$.
What does this tell us about $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$?
Draw a picture to illustrate this fact (sketch $\mathbf{u}$, $\mathbf{v}$, $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{v}$ with labels).