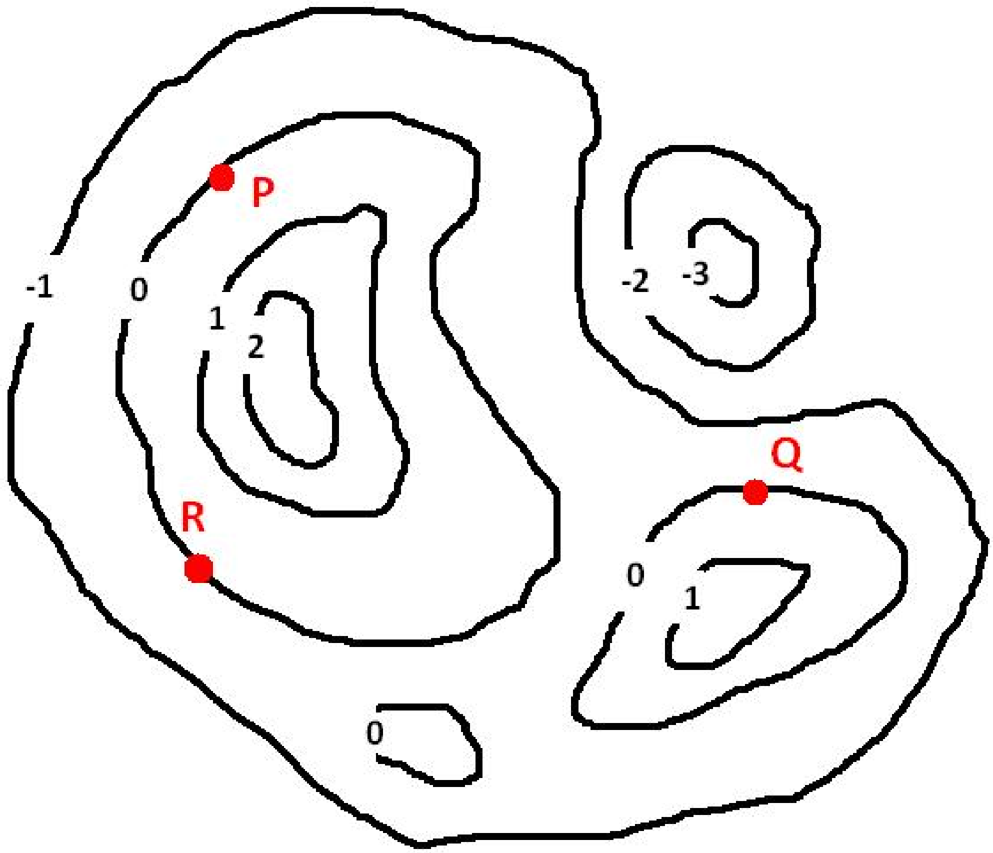
Math 2130-101 **Test #2 for Section 101** **October 14^{th}, 2009**

Name: ________________________________ **Be sure to show your work!**

1. (____/10 points) Compute the curvature of $\mathbf{r}(t) = (t+2, 3t+4, 5t+6)$.

2. (____/10 points) Let $f(x,y) = x + x^2y^2 - y$. Find the equation of the line tangent to $f(x,y) = 1$ at the point $(-1, 2)$.

3. (____/10 points) The following graph is a contour map of $z = f(x, y)$. Each contour is labeled with its z-value (i.e. "height").



(a) For each of the following partials derivatives, use the contour plot to decide whether they are positive, negative, or zero.

i. $f_x(P)$ is ________________ .

ii. $f_y(P)$ is ________________ .

iii. $f_x(Q)$ is ________________ .

iv. $f_y(Q)$ is ________________ .

(b) Sketch $\nabla f(R)$ in the plot above.

4. (____/12 points) Find the quadratic approximation of $f(x, y) = x^2 y$ at the point $(-1, 1)$.

5. (____/12 points) Let $f(x, y) = e^{x+y}$.

(a) Compute $\mathbf{D_u} f(1, -1)$ where $\mathbf{u} = \frac{1}{\sqrt{2}}(-1, 1)$.

(b) If I want to maximize $\mathbf{D_w} f(1, -1)$, what vector $\mathbf{w}$ should I use?

6. (____/12 points) Let $f(x, y) = (xy, y^2)$ and $g(u, v) = 2u - v$.

(a) Find the Jacobian, f', of f.

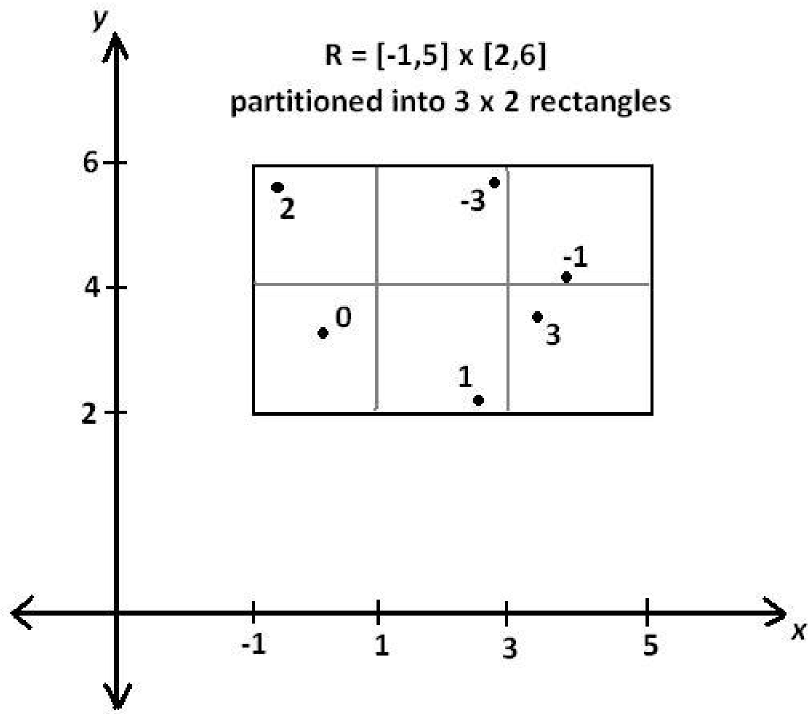
(b) Find the Jacobian, g', of g.

(c) Use the chain rule to find the Jacobian, $(g \circ f)'$, of $g \circ f$.

7. (____/12 points) Find the maximum and minimum values of $f(x,y) = 2x - 6y$ if $x^2 + 3y^2 = 1$.

8. (____/12 points) The function $f(x,y) = 4xy - x^4 - y^4$ has critical points located at $(0,0)$, $(1,1)$, and $(-1,-1)$. Determine whether each point is a relative minimum, relative maximum, or saddle point.

9. (____/10 points) Approximate the integral $\iint_R f(x,y)\,dA$ where R, a partition, and sample points with their $f(x,y)$-values are shown below.



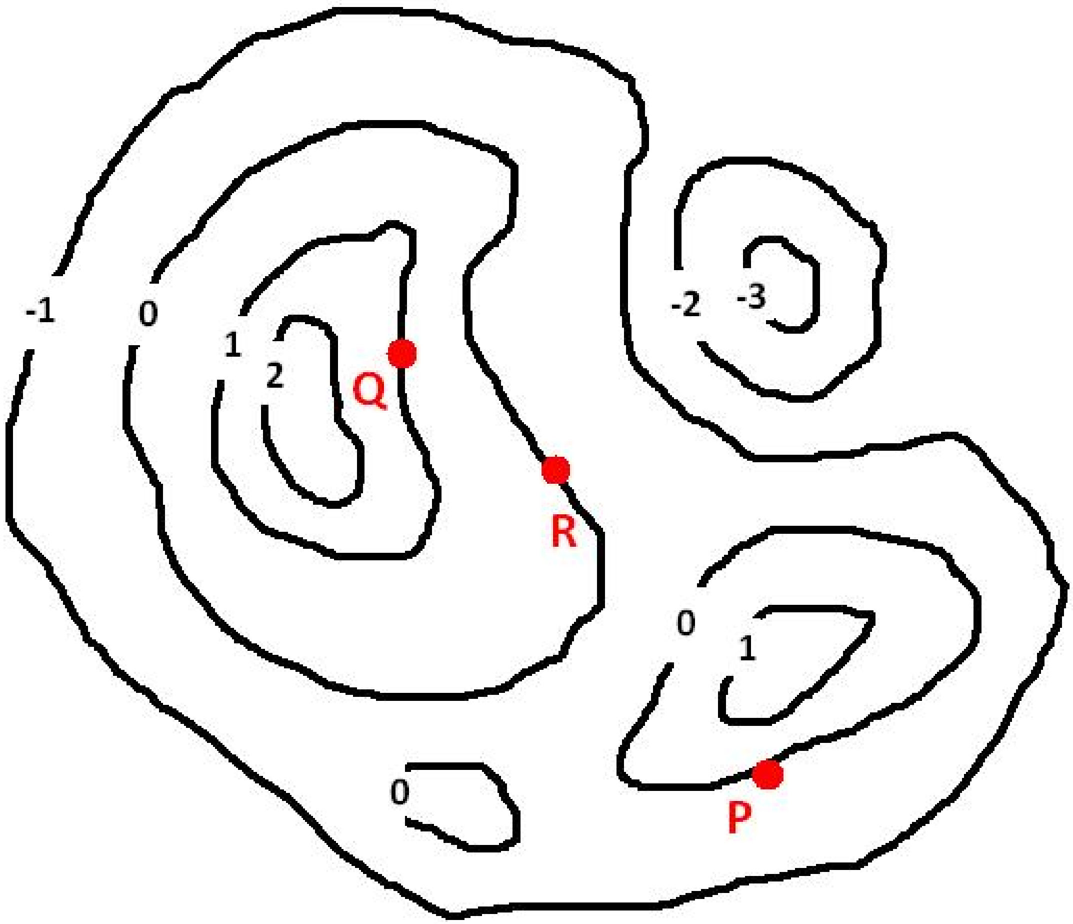
Math 2130-102 **Test #2 for Section 102** **October 14^{th}, 2009**

Name: ________________________________ **Be sure to show your work!**

1. (____/10 points) Compute the curvature of $\mathbf{r}(t) = (2\cos(t) + 3, 2\sin(t) + 4)$.

2. (____/12 points) Let $f(x, y, z) = x^2 + 2y^2 + 3z^2$. Find the equation of the plane tangent to $f(x, y, z) = 7$ at the point $(2, 0, 1)$.

3. (____/10 points) The following graph is a contour map of $z = f(x, y)$. Each contour is labeled with its z-value (i.e. "height").



(a) For each of the following partials derivatives, use the contour plot to decide whether they are positive, negative, or zero.

i. $f_x(P)$ is ______________ .

ii. $f_y(P)$ is ______________ .

iii. $f_x(Q)$ is ______________ .

iv. $f_y(Q)$ is ______________ .

(b) Sketch $\nabla f(R)$ in the plot above.

4. (____/12 points) Find the quadratic approximation of $f(x, y) = xy^2$ at the point $(0, -1)$.

5. (____/12 points) Let $f(x, y, z) = xyz$.

(a) Compute $\mathbf{D_u}f(1, 0, -1)$ where $\mathbf{u} = \frac{1}{\sqrt{2}}(0, 1, -1)$.

(b) What is the maximum possible **value** of $\mathbf{D_w}f(1, 0, -1)$?

6. (____/12 points) Let $f(x, y) = (xy, x^2, 2x - y)$.

(a) Find the Jacobian, f', of f.

(b) Find the linearization of f at $(1, 0)$.

7. (____/10 points) Set up equations (coming from the Lagrange multiplier method) which allow you to find the maximum and minimum value of $f(x, y) = 2xy$ subject to the constraint $x^2 + y^2 = 2$. **Just set up the equations — don't solve.**

8. (____/12 points) Find the critical points of $f(x, y) = x^2 + 2y^2 + xy^2 + 1$ and then determine whether each is a relative maximum, relative minimum or saddle point. **Hint:** $4y + 2xy = 2 \cdot y \cdot (x + 2)$.

9. (____/10 points) Use the **midpoint rule** to approximate the integral $\iint_R 2x + y\,dA$ where R and its partition are shown below.