Name: _____   **Be sure to show your work!**

1. (____/16 points) Consider the function $f(x, y) = \begin{cases} x^{-2}y^{-2} & x \geq 1 \text{ and } y \geq 1 \\ 0 & \text{otherwise} \end{cases}$

It's easy to see that $f(x, y) \geq 0$ everywhere. Compute $\displaystyle\iint_{\mathbb{R}^2} f(x, y)\, dA$ and decide if f is a probability distribution function.

Is f a probability distribution function?   YES   /   NO

2. (____/16 points) Let R be the region bounded by $y = x^2$ and $y = 18 - x^2$. Write the following integral as an iterated integral in **BOTH** orders of integration:

$$\iint_R x^2 e^{xy}\, dA$$

You don't need to evaluate these integrals.
Hint: You may need to split one of your integrals into two pieces.

3. (____/**16 points**) Consider the interated integral:

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} \sqrt{x^2+y^2+z^2} \, dz \, dy \, dx$$

(a) Rewrite the integral with the order integration $\iiint dx \, dz \, dy$.
   You don't need to evaluate this integral.

(b) Convert the integral to cylindrical coordinates.
   You don't need to evaluate this integral.

(c) Convert the integral to spherical coordinates.
   You don't need to evaluate this integral.

4. (____/20 points) Let R be the region bounded by $y = 0$, $x = 0$, and $x + y = 2$. Evaluate the integral

$$\iint_R 3 \left(\frac{x - y}{x + y} \right)^2 dA$$

by changing coordinates using the transformation $u = x + y$ and $v = x - y$.

5. (_____/**16 points**) Let E be the region below $z = 4$ and above $z = x^2 + y^2$.
Find the centroid of E.   **Free Information:** The volume of E is 8π.

6. (____/16 points) Let E be the region inside $x^2 + y^2 + z^2 = 4$ and outside $x^2 + y^2 + z^2 = 1$. Evaluate the integral:

$$\iiint_E 3e^{(x^2+y^2+z^2)^{3/2}} \, dV$$

Name: _____   **Be sure to show your work!**

1. (____/20 points) Consider the function $f(x, y) = e^{-x^2 - y^2}$. It's easy to see that $f(x,y) \geq 0$ every-where. Compute $\displaystyle\iint_{\mathbb{R}^2} f(x,y)\, dA$ and decide if f is a probability distribution function.

Is f a probability distribution function?   YES   /   NO

2. (____/20 points) Reverse the order of integration and then evaluate the integral

$$\int_0^2 \int_{x/2}^1 e^{\left(y^2\right)} \, dy \, dx$$

3. (_____/**20 points**) Let E be the region inside the cylinder $x^2 + y^2 = 1$, above the xy-plane and below
the plane $z = 3$ Hey, I'm a cylinder!

(a) Write $\iiint_E \sqrt{x^2 + y^2}\, dV$ as an iterated integral with the following order of integration: $\int\int\int dz\, dy\, dx$.
Do not evaluate this integral.

(b) Write $\iiint_E \sqrt{x^2 + y^2}\, dV$ as an iterated integral with the following order of integration: $\int\int\int dx\, dz\, dy$.
Do not evaluate this integral.

(c) Convert $\iiint_E \sqrt{x^2 + y^2}\, dV$ to cylindrical coordinates and then **evaluate the integral**.

4. (____/**20 points**) Let R be the region bounded by $x + y = 1$, $x - y = 0$, and $y = 0$. Let $u = x + y$ and $v = x - y$. Use this change of variables to evaluate $\iint_R (x + y)e^{x-y}\, dA$.

5. (_____/**20 points**) Find the centroid of the E where E the region inside $x^2 + y^2 + z^2 = 4$ and above the xy-plane. *Hint:* Symmetry + Geometry = Only 1 integral to compute.