Math 2130-101

Test #1

September 21st, 2011

Name: Answer Key

Be sure to show your work!

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u}\bullet\mathbf{v}}{|\mathbf{v}|^2}\mathbf{v}$$

$$\kappa = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$\mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t)\bullet\mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t)$$

$$\kappa = \frac{|f''(x)|}{(1+(f'(x))^2)^{\frac{3}{2}}}$$

1. (15 points) Let $\mathbf{u} = \langle 1,-2,2\rangle$, $\mathbf{v} = \langle 1,1,-1\rangle$, and $\mathbf{w} = \langle 0,-1,-1\rangle$.

(a) Find **both** of the unit vectors which are parallel to $\mathbf{u}$

If we normalize $\mathbf{u}$, we'll have a vector pointing in the same direction as $\mathbf{u}$ but is of length 1. The other unit vector parallel to $\mathbf{u}$ would need to point in the opposite direction. To get this vector, we simply multiply the normalization by -1.

Answer: $\pm\frac{\mathbf{u}}{|\mathbf{u}|} = \pm\frac{1}{\sqrt{1^2+(-2)^2+2^2}}\langle 1,-2,2\rangle = \pm\frac{1}{\sqrt{9}}\langle 1,-2,2\rangle = \pm\left\langle \frac{1}{3},-\frac{2}{3},\frac{2}{3}\right\rangle$

(b) Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

This is given by the absolute value of the triple scalar product of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$. We can either compute the determinant of the 3×3 matrix which has rows given by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ or we can compute a cross product and then a dot product. I'll do the later.

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 2 \\ 1 & 1 & -1 \end{vmatrix} = ((-2)(-1)-(1)(2))\mathbf{i} - ((1)(-1)-(1)(2))\mathbf{j} + ((1)(1)-(1)(-2))\mathbf{k} = \langle 0,3,3\rangle$$

So the volume is $|(\mathbf{u}\times\mathbf{v})\bullet w| = |\langle 0,3,3\rangle\bullet\langle 0,-1,-1\rangle| = |-6| = 6$.

(c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ (don't worry about evaluating inverse trigonometric functions).

We know that $\mathbf{u}\bullet\mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos(\theta)$ where θ is the angle between the vectors. Note that $|\mathbf{u}| = \sqrt{1^2+(-2)^2+2^2} = \sqrt{9} = 3$, $|\mathbf{v}| = \sqrt{1^2+1^2+(-1)^2} = \sqrt{3}$, and $\mathbf{u}\bullet\mathbf{v} = (1)(1)+(-2)(1)+(2)(-1) = -3$.

The angle between $\mathbf{u}$ and $\mathbf{v}$ is $\theta = \arccos\left(\frac{\mathbf{u}\bullet\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right) = \arccos\left(\frac{-3}{3\sqrt{3}}\right) = \arccos\left(-\frac{1}{\sqrt{3}}\right)$

Is this angle... **right,** **acute,** or [✓ **obtuse**] ? (Circle your answer.) Because the dot product is negative.

2. (14 points) Lines!

(a) Find parametric equations (or a vector valued function) for the line which passes through the point $P = (2,-1,3)$ and is parallel to the line $\mathbf{r}(t) = \langle 2t-4,-t+3,3t+1\rangle$.

A parallel line's direction vector should be parallel to the direction vector of $\mathbf{r}(t)$ – that is – $\mathbf{r}'(t) = \langle 2,-1,3\rangle$. Therefore, $\mathbf{r}(t) = \langle 2,-1,3\rangle + \langle 2,-1,3\rangle t$ is parallel to the first line and passes through the point $(2,-1,3)$.

(b) Let ℓ_1 be the line parametrized by $\mathbf{r}_1(t) = \langle 2t+1,4t,-2t-1\rangle$ and ℓ_2 be the line parametrized by $\mathbf{r}_2(t) = \langle -t-1,-2t+1,t+1\rangle$. Determine if ℓ_1 and ℓ_2 are the same, parallel, intersecting, or skew.

$\mathbf{r}_1'(t) = \langle 2,4,-2\rangle$ and $\mathbf{r}_2'(t) = \langle -1,-2,1\rangle$. Notice that $-2\cdot\mathbf{r}_2'(t) = \mathbf{r}_1'(t)$. Therefore, these lines have parallel direction vectors (thus they are either parallel lines or the same line parametrized two different ways). Let's see if they intersect.

$\mathbf{r}_1(t) = \mathbf{r}_2(s)$ implies that $2t+1 = -s-1$, $4t = -2s+1$, and $-2t-1 = s+1$. The last equation says that $s = -2t-2$. Plugging this into the second equation gives us $4t = -2(-2t-2)+1 = 4t+5$. So that $0 = 5$. But this is impossible. Therefore, there is no solution. These lines do not intersect (so they are not the same line).

Answer: These are parallel lines.

3. (10 points) A "plane" old problem:
Find an equation for the plane which passes through the points $(1,3,3)$, $(2,1,-1)$ and is parallel to the vector $\mathbf{v} = \langle 1,0,2\rangle$.

We need a point and a normal vector. We've been given two points (either will do just fine). The difference of these two points will give us a vector which is parallel to the plane: $\mathbf{w} = \langle 2-1, 1-3, -1-3\rangle = \langle 1,-2,-4\rangle$. Since $\mathbf{w}$ and $\mathbf{v}$ are parallel to the plane, their cross product will give us a normal vector.

$$\mathbf{w}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & -4 \\ 1 & 0 & 2 \end{vmatrix} = ((-2)(2)-(0)(-4))\mathbf{i} - ((1)(2)-(1)(-4))\mathbf{j} + ((1)(0)-(1)(-2))\mathbf{k} = \langle -4,-6,2\rangle$$

Answer: $(-4)(x-1) + (-6)(y-3) + (2)(z-3) = 0$

4. (10 points) Parametrize $(x-2)^2 + (y+3)^2 = 5^2$.
Call your parameter t and don't forget to give t's domain: $?a? \leq t \leq ?b?$.

Answer: $\mathbf{r}(t) = \langle 5\cos(t)+2, 5\sin(t)-3\rangle$, $0 \leq t \leq 2\pi$ (Of course, this is just one of infinitely many correct answers.)

5. (10 points) Set up an integral which computes the arc length of the curve $\mathbf{r}(t) = \langle 3\sin(t), 5t, \cos(t)\rangle$ where $-2 \leq t \leq 5$ [If you're curious, this is part of an elliptic helix].

Do not attempt to evaluate this integral (it will just end in tears).

$\mathbf{r}'(t) = \langle 3\cos(t), 5, -\sin(t)\rangle$ and so $|\mathbf{r}'(t)| = \sqrt{9\cos^2(t) + 25 + \sin^2(t)}$

Answer: Arc Length $= \displaystyle\int_{-2}^{5} \sqrt{9\cos^2(t)+25+\sin^2(t)}\,dt$

Note: The above integral cannot be evaluated by hand. The answer involves "elliptic functions."

6. (15 points) Find the **TNB**-frame for $\mathbf{r}(t) = \langle 3\sin(t), 5, 3\cos(t)\rangle$.

$\mathbf{r}'(t) = \langle 3\cos(t), 0, -3\sin(t)\rangle$ and so $|\mathbf{r}'(t)| = \sqrt{9\cos^2(t) + 0^2 + 9\sin^2(t)} = \sqrt{9} = 3$. Therefore,

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \langle \cos(t), 0, -\sin(t)\rangle$$

$\mathbf{T}'(t) = \langle -\sin(t), 0, -\cos(t)\rangle$ and so $|\mathbf{T}'(t)| = \sqrt{\sin^2(t) + 0^2 + \cos^2(t)} = 1$. Therefore,

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \langle -\sin(t), 0, -\cos(t)\rangle$$

Finally,

$$\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos(t) & 0 & -\sin(t) \\ -\sin(t) & 0 & -\cos(t) \end{vmatrix} = \langle 0, -(-\cos^2(t) - \sin^2(t)), 0\rangle = \langle 0,1,0\rangle$$

7. (14 points) Curvature.

(a) Find a formula for the curvature of $\mathbf{r}(t) = \big\langle \sin(t), t^2, \cos(t)\big\rangle$.

Unless you have a curve of the form $y = f(x)$ or unless you have already computed $\mathbf{T}$, the easiest formula for computing curvature is the one with the cross product. Note that $\mathbf{r}'(t) = \langle \cos(t), 2t, -\sin(t)\rangle$, $\mathbf{r}''(t) = \langle -\sin(t), 2, -\cos(t)\rangle$, and $|\mathbf{r}'(t)| = \sqrt{\cos^2(t) + (2t)^2 + \sin^2(t)} = \sqrt{1+4t^2}$ Next,

$$\mathbf{r}'(t)\times\mathbf{r}''(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos(t) & 2t & -\sin(t) \\ -\sin(t) & 2 & -\cos(t) \end{vmatrix} = \langle -2t\cos(t) + 2\sin(t), -(-\cos^2(t) - \sin^2(t)), 2\cos(t) + 2t\sin(t)\rangle$$

Thus $\mathbf{r}'(t)\times\mathbf{r}''(t) = \langle -2t\cos(t) + 2\sin(t), 1, 2\cos(t) + 2t\sin(t)\rangle$ and so

$$\kappa(t) = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{\sqrt{(-2t\cos(t)+2\sin(t))^2 + 1 + (2\cos(t)+2t\sin(t))^2}}{(1+4t^2)^{3/2}}$$

(I didn't promise the answer would be pretty.)

(b) Find a formula for the curvature of $y = x^2$. Then find where curvature is maximized. [*Hint:* Sketch the graph of $y = x^2$. Does you answer make sense?]

Since we have a function of the form $y = f(x)$, our special formula applies:

$$\kappa(x) = \frac{|f''(x)|}{(1 + (f'(x))^2)^{3/2}} = \frac{2}{(1 + 4x^2)^{3/2}}$$

This function is maximized when $x = 0$ (the smaller the denominator the bigger the fraction). This makes good sense since $x = 0$ is the location of the vertex of the parabola $y = x^2$ and that is where a parabola is "bent" the most.

8. (12 points) No numbers here.

(a) Choose **ONE** of the following:

I. Show why $(\mathbf{a} \bullet \mathbf{b})^2 + |\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2$.
[Don't try to do this in terms of components. Instead. use dot and cross product identities.]

II. Suppose the vector valued function $\mathbf{r}(t)$ gives the position of an object. [Which functions give velocity, acceleration, and speed?] Suppose $\mathbf{r}(t)$ has **constant** speed. Why are the velocity and acceleration vectors orthogonal?

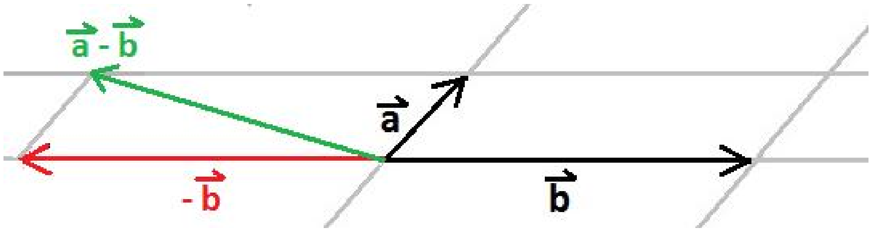
I. $(\mathbf{a} \bullet \mathbf{b})^2 + |\mathbf{a} \times \mathbf{b}|^2 = (|\mathbf{a}||\mathbf{b}|\cos(\theta))^2 + (|\mathbf{a}||\mathbf{b}|\sin(\theta))^2 = |\mathbf{a}|^2|\mathbf{b}|^2(\sin^2(t) + \cos^2(t)) = |\mathbf{a}|^2|\mathbf{b}|^2$

II. If $\mathbf{r}(t)$ has constant speed, then $|\mathbf{r}'(t)| = c$ for some constant c. We know that if $\mathbf{v}(t)$ is a constant length vector valued function, then $\mathbf{v}(t)$ and $\mathbf{v}'(t)$ are orthogonal. Thus $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ are orthgonal. Alternatively,

$$0 = \frac{d}{dt}\left[c^2\right] = \frac{d}{dt}\left[|\mathbf{r}'(t)|^2\right] = \frac{d}{dt}\left[\mathbf{r}'(t) \bullet \mathbf{r}'(t)\right] = \mathbf{r}''(t) \bullet \mathbf{r}'(t) + \mathbf{r}'(t) \bullet \mathbf{r}''(t) = 2\mathbf{r}'(t) \bullet \mathbf{r}''(t)$$

Thus $\mathbf{r}'(t) \bullet \mathbf{r}''(t) = 0$ and so velocity and acceleration are orthongonal.

(b) $\mathbf{a}$ and $\mathbf{b}$ are pictured below. Sketch $-\mathbf{b}$ and $\mathbf{a} - \mathbf{b}$.



Math 2130-102

Test #1

September 21st, 2011

Name: Answer Key

Be sure to show your work!

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v}$$

$$\kappa = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$\mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t)$$

$$\kappa = \frac{|f''(x)|}{(1+(f'(x))^2)^{\frac{3}{2}}}$$

1. (15 points) Let $\mathbf{u} = \langle 2, 0, -1\rangle$, $\mathbf{v} = \langle 2, 1, 3\rangle$, and $\mathbf{w} = \langle 1, 1, 0\rangle$.

(a) Find **both** of the unit vectors which are parallel to $\mathbf{u}$

If we normalize $\mathbf{u}$, we'll have a vector pointing in the same direction as $\mathbf{u}$ but is of length 1. The other unit vector parallel to $\mathbf{u}$ would need to point in the opposite direction. To get this vector, we simply multiply the normalization by -1.

Answer: $\pm\dfrac{\mathbf{u}}{|\mathbf{u}|} = \pm\dfrac{1}{\sqrt{2^2+0^2+(-1)^2}}\langle 2, 0, -1\rangle = \pm\dfrac{1}{\sqrt{5}}\langle 2, 0, -1\rangle$

(b) Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

This is given by the absolute value of the triple scalar product of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$. We can either compute the determinant of the 3×3 matrix which has rows given by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ or we can compute a cross product and then a dot product. I'll do the later.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & -1 \\ 2 & 1 & 3 \end{vmatrix} = ((0)(3)-(1)(-1))\mathbf{i} - ((2)(3)-(2)(-1))\mathbf{j} + ((2)(1)-(2)(0))\mathbf{k} = \langle 1, -8, 2\rangle$$

So the volume is $|(\mathbf{u} \times \mathbf{v}) \bullet w| = |\langle 1, -8, 2\rangle \bullet \langle 1, 1, 0\rangle| = |-7| = 7$.

(c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ (don't worry about evaluating inverse trigonometric functions).

We know that $\mathbf{u} \bullet \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos(\theta)$ where θ is the angle between the vectors. Note that $|\mathbf{u}| = \sqrt{2^2+0^2+(-1)^2} = \sqrt{5}$, $|\mathbf{v}| = \sqrt{2^2+1^2+3^2} = \sqrt{14}$, and $\mathbf{u} \bullet \mathbf{v} = (2)(2)+(0)(1)+(-1)(3) = 1$.

The angle between $\mathbf{u}$ and $\mathbf{v}$ is $\theta = \arccos\left(\dfrac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right) = \arccos\left(\dfrac{1}{\sqrt{5}\sqrt{14}}\right) = \arccos\left(\dfrac{1}{\sqrt{70}}\right)$

Is this angle... **right**, [✓ **acute**], or **obtuse** ? (Circle your answer.) Because the dot product is positive.

2. (14 points) Lines!

(a) Find parametric equations (or a vector valued function) for the line which passes through the point $P = (1, 3, -2)$ and is parallel to the line $\mathbf{r}(t) = \langle t+5, 2t-2, -t+3\rangle$.

A parallel line's direction vector should be parallel to the direction vector of $\mathbf{r}(t)$ – that is – $\mathbf{r}'(t) = \langle 1, 2, -1\rangle$. Therefore, $\mathbf{r}(t) = \langle 1, 3, -2\rangle + \langle 1, 2, -1\rangle t$ is parallel to the first line and passes through the point $(1, 3, -2)$.

(b) Let ℓ_1 be the line parametrized by $\mathbf{r}_1(t) = \langle t+1, -2t, t+2\rangle$ and ℓ_2 be the line parametrized by $\mathbf{r}_2(t) = \langle -2t+1, 4t+2, -2t+1\rangle$. Determine if ℓ_1 and ℓ_2 are the same, parallel, intersecting, or skew.

$\mathbf{r}_1'(t) = \langle 1, -2, 1\rangle$ and $\mathbf{r}_2'(t) = \langle -2, 4, -2\rangle$. Notice that $-2 \cdot \mathbf{r}_1'(t) = \mathbf{r}_2'(t)$. Therefore, these lines have parallel direction vectors (thus they are either parallel lines or the same line parametrized two different ways). Let's see if they intersect.

$\mathbf{r}_1(t) = \mathbf{r}_2(s)$ implies that $t+1 = -2s+1$, $-2t = 4s+2$, and $t+2 = -2s+1$. The last equation says that $t = -2s-1$. Plugging this into the second equation gives us $-2(-2s-1) = 4s+2$ and so $4s+2 = 4s+2$ so that $0 = 0$. Finally, the first equation says, $(-2s-1)+1 = -2s+1$ and so $-2s = -2s+1$ thus $0 = 1$. This is impossible. Therefore, there is no solution. These lines do not intersect (so they are not the same line).

Answer: These are parallel lines.

3. (12 points) A “plane” old problem:
Find an equation for the plane which passes through the points $(1,1,1)$, $(1,2,3)$, and $(3,2,1)$.

We need a point (any of the three above will do just fine) and a normal vector. We can find a normal vector by cross producting two vectors which are parallel to the plane. To get two vectors parallel to the plane we can take differences of points on the plane.

Thus $\mathbf{v} = \langle 1-1, 2-1, 3-1\rangle = \langle 0,1,2\rangle$ and $\mathbf{w} = \langle 3-1, 2-1, 1-1\rangle = \langle 2,1,0\rangle$ are parallel to the plane.

$$\mathbf{v}\times\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 2 \\ 2 & 1 & 0 \end{vmatrix} = ((1)(0)-(1)(2))\mathbf{i} - ((0)(0)-(2)(2))\mathbf{j} + ((0)(1)-(2)(1))\mathbf{k} = \langle -2, 4, -2\rangle$$

Answer: $(-2)(x-1)+(4)(y-1)+(-2)(z-1)=0$

4. (10 points) Set up an integral which computes the arc length of the curve $\mathbf{r}(t) = \langle t^2, \cos(t), 4\sin(t)\rangle$ where $-2 \leq t \leq 4$. **Do not** attempt to evaluate this integral (it will just end in tears).

$\mathbf{r}'(t) = \langle 2t, -\sin(t), 4\cos(t)\rangle$ and so $|\mathbf{r}'(t)| = \sqrt{4t^2+\sin^2(t)+16\cos^2(t)}$

Answer: Arc Length $= \displaystyle\int_{-2}^{4} \sqrt{4t^2+\sin^2(t)+16\cos^2(t)}\,dt$

5. (15 points) **TNB**-Frames.

(a) Find the **TNB**-frame for $\mathbf{r}(t) = \langle 3, 2\cos(t), 2\sin(t)\rangle$.

$\mathbf{r}'(t) = \langle 0, -2\sin(t), 2\cos(t)\rangle$ and so $|\mathbf{r}'(t)| = \sqrt{0^2+4\sin^2(t)+4\cos^2(t)} = \sqrt{4} = 2$. Therefore,

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \langle 0, -\sin(t), \cos(t)\rangle$$

$\mathbf{T}'(t) = \langle 0, -\cos(t), -\sin(t)\rangle$ and so $|\mathbf{T}'(t)| = \sqrt{0^2+\cos^2(t)+\sin^2(t)} = 1$. Therefore,

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \langle 0, -\cos(t), -\sin(t)\rangle$$

Finally,

$$\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -\sin(t) & \cos(t) \\ 0 & -\cos(t) & -\sin(t) \end{vmatrix} = \langle \sin^2(t)+\cos^2(t), 0, 0\rangle = \langle 1,0,0\rangle$$

(b) If $\mathbf{B}(t)$ is a constant vector, what can we conclude?

If the binormal is constant, then the tangent and normal vectors span the same (osculating) plane everywhere. This means that the curve lies in a plane.
Answer: The curve in planar (it lies in a plane).

6. (12 points) Find a formula for the curvature of $\mathbf{r}(t) = \left\langle \sin(t), t^2, t^3\right\rangle$.

Unless you have a curve of the form $y = f(x)$ or unless you have already computed $\mathbf{T}$, the easiest formula for computing curvature is the one with the cross product. Note that $\mathbf{r}'(t) = \langle \cos(t), 2t, 3t^2\rangle$, $\mathbf{r}''(t) = \langle -\sin(t), 2, 6t\rangle$, and $|\mathbf{r}'(t)| = \sqrt{\cos^2(t)+4t^2+9t^4}$ Next,

$$\mathbf{r}'(t)\times\mathbf{r}''(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos(t) & 2t & 3t^2 \\ -\sin(t) & 2 & 6t \end{vmatrix} = \langle 12t^2-6t^2, -6t\cos(t)-3t^2\sin(t), 2\cos(t)+2t\sin(t)\rangle$$

Thus $\mathbf{r}'(t)\times\mathbf{r}''(t) = \langle 6t^2, -6t\cos(t)-3t^2\sin(t), 2\cos(t)+2t\sin(t)\rangle$ and so

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{\sqrt{36t^4 + (6t\cos(t) + 3t^2\sin(t))^2 + (2\cos(t) + 2t\sin(t))^2}}{(\cos^2(t) + 4t^2 + 9t^4)^{3/2}}$$

(I didn't promise the answer would be pretty.)

7. (12 points) No numbers here.

(a) Choose **ONE** of the following:

I. Show why $(\mathbf{a} \bullet \mathbf{b})^2 + |\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2$.
[Don't try to do this in terms of components. Instead. use dot and cross product identities.]

II. Suppose the vector valued function $\mathbf{r}(t)$ gives the position of an object. [Which functions give velocity, acceleration, and speed?] Suppose $\mathbf{r}(t)$ has **constant** speed. Why are the velocity and acceleration vectors orthogonal?

See Section 101's answer key.

(b) **a** and **b** are pictured below. Sketch $-\mathbf{a}$ and $\mathbf{b} - \mathbf{a}$.