Math 2130-101 **Test #1** **September 21^{st}, 2011**

Name: ______________________________ **Be sure to show your work!**

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} \qquad \kappa = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$\mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t) \qquad \kappa = \frac{|f''(x)|}{(1+(f'(x))^2)^{\frac{3}{2}}}$$

1. (____/15 points) Let $\mathbf{u} = \langle 1, -2, 2\rangle$, $\mathbf{v} = \langle 1, 1, -1\rangle$, and $\mathbf{w} = \langle 0, -1, -1\rangle$.

(a) Find **both** of the unit vectors which are parallel to $\mathbf{u}$

(b) Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

(c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ (don't worry about evaluating inverse trigonometric functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

2. (____/14 points) Lines!

(a) Find parametric equations (or a vector valued function) for the line which passes through the point $P = (2, -1, 3)$ and is parallel to the line $\mathbf{r}(t) = \langle 2t - 4, -t + 3, 3t + 1 \rangle$.

(b) Let ℓ_1 be the line parametrized by $\mathbf{r}_1(t) = \langle 2t + 1, 4t, -2t - 1 \rangle$ and ℓ_2 be the line parametrized by $\mathbf{r}_2(t) = \langle -t - 1, -2t + 1, t + 1 \rangle$. Determine if ℓ_1 and ℓ_2 are the same, parallel, intersecting, or skew.

3. (____/10 points) A "plane" old problem:
Find an equation for the plane which passes through the points $(1, 3, 3)$, $(2, 1, -1)$ and is parallel to the vector $\mathbf{v} = \langle 1, 0, 2 \rangle$.

4. (____/10 points) Parametrize $(x-2)^2 + (y+3)^2 = 5^2$.
Call your parameter t and don't forget to give t's domain: $?a? \leq t \leq ?b?$.

5. (____/10 points) Set up an integral which computes the arc length of the curve $\mathbf{r}(t) = \langle 3\sin(t), 5t, \cos(t)\rangle$ where $-2 \leq t \leq 5$ [If you're curious, this is part of an elliptic helix].

Do not attempt to evaluate this integral (it will just end in tears).

6. (____/15 points) Find the **TNB**-frame for $\mathbf{r}(t) = \langle 3\sin(t), 5, 3\cos(t)\rangle$.

7. (____/14 points) Curvature.

(a) Find a formula for the curvature of $\mathbf{r}(t) = \langle \sin(t), t^2, \cos(t) \rangle$.

(b) Find a formula for the curvature of $y = x^2$. Then find where curvature is maximized. [*Hint:* Sketch the graph of $y = x^2$. Does you answer make sense?]

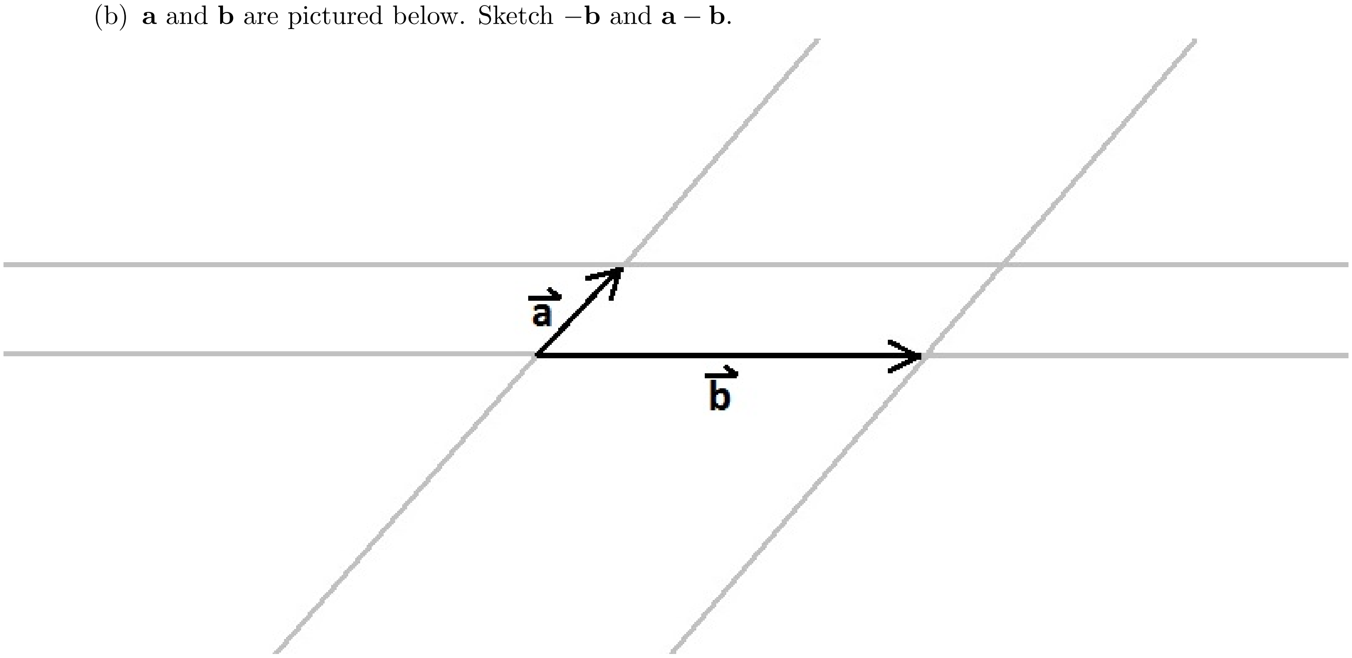
8. (____/12 points) No numbers here.

(a) Choose **ONE** of the following:

I. Show why $(\mathbf{a} \bullet \mathbf{b})^2 + |\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2$.
[Don't try to do this in terms of components. Instead. use dot and cross product identities.]

II. Suppose the vector valued function $\mathbf{r}(t)$ gives the position of an object. [Which functions give velocity, acceleration, and speed?] Suppose $\mathbf{r}(t)$ has **constant** speed. Why are the velocity and acceleration vectors orthogonal?

(b) $\mathbf{a}$ and $\mathbf{b}$ are pictured below. Sketch $-\mathbf{b}$ and $\mathbf{a} - \mathbf{b}$.



Math 2130-102 **Test #1** **September 21^{st}, 2011**

Name: ______________________________ **Be sure to show your work!**

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} \qquad \kappa = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$\mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t) \qquad \kappa = \frac{|f''(x)|}{(1+(f'(x))^2)^{\frac{3}{2}}}$$

1. (____/15 points) Let $\mathbf{u} = \langle 2, 0, -1 \rangle$, $\mathbf{v} = \langle 2, 1, 3 \rangle$, and $\mathbf{w} = \langle 1, 1, 0 \rangle$.

(a) Find **both** of the unit vectors which are parallel to $\mathbf{u}$

(b) Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

(c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ (don't worry about evaluating inverse trigonometric functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

2. (____/14 points) Lines!

(a) Find parametric equations (or a vector valued function) for the line which passes through the point $P = (1, 3, -2)$ and is parallel to the line $\mathbf{r}(t) = \langle t + 5, 2t - 2, -t + 3\rangle$.

(b) Let ℓ_1 be the line parametrized by $\mathbf{r}_1(t) = \langle t + 1, -2t, t + 2\rangle$ and ℓ_2 be the line parametrized by $\mathbf{r}_2(t) = \langle -2t + 1, 4t + 2, -2t + 1\rangle$. Determine if ℓ_1 and ℓ_2 are the same, parallel, intersecting, or skew.

3. (____/12 points) A "plane" old problem:
Find an equation for the plane which passes through the points $(1, 1, 1)$, $(1, 2, 3)$, and $(3, 2, 1)$.

4. (____/10 points) Set up an integral which computes the arc length of the curve $\mathbf{r}(t) = \langle t^2, \cos(t), 4\sin(t)\rangle$ where $-2 \leq t \leq 4$.

Do not attempt to evaluate this integral (it will just end in tears).

5. (____/15 points) **TNB**-Frames.

(a) Find the **TNB**-frame for $\mathbf{r}(t) = \langle 3, 2\cos(t), 2\sin(t)\rangle$.

(b) If $\mathbf{B}(t)$ is a constant vector, what can we conclude?

6. (____/12 points) Find a formula for the curvature of $\mathbf{r}(t) = \left\langle \sin(t), t^2, t^3 \right\rangle$.

7. (____/12 points) No numbers here.

(a) Choose **ONE** of the following:

I. Show why $(\mathbf{a} \bullet \mathbf{b})^2 + |\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2$.
[Don't try to do this in terms of components. Instead. use dot and cross product identities.]

II. Suppose the vector valued function $\mathbf{r}(t)$ gives the position of an object. [Which functions give velocity, acceleration, and speed?] Suppose $\mathbf{r}(t)$ has **constant** speed. Why are the velocity and acceleration vectors orthogonal?

(b) $\mathbf{a}$ and $\mathbf{b}$ are pictured below. Sketch $-\mathbf{a}$ and $\mathbf{b} - \mathbf{a}$.