Math 2130-101 | # Test #2 | **October 24^{th}, 2011**

Name: ________________________________ **Be sure to show your work!**

If $F(x,y)=C$, then $\dfrac{dy}{dx}=-\dfrac{F_x}{F_y}$ If $F(x,y,z)=C$, then $\dfrac{\partial z}{\partial x}=-\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y}=-\dfrac{F_y}{F_z}$

1. (____/9 points) Consider the surface $z=\sqrt{x^2+y^2}$.

(a) Write down an equation for the level curves of this surface. What are these curves?

(b) Write down an equation for the trace made by intersecting this surface with the yz-plane.

(c) Make a rough sketch of this surface.

2. (____/10 points) Show that the limit $\displaystyle\lim_{(x,y)\to(0,0)}\frac{2x^2y}{x^4+y^2}$ does not exist. *Hint:* Consider $y=x^2$.

3. (_____/9 points) Suppose $xz - e^{yz} = 5$. Considering x and y as independent variables and z as a dependent variable, compute the implicit derivative $\dfrac{\partial z}{\partial y}$

4. (_____/12 points) Let $f(x, y) = x^4 + 4xy + 4y - 1$

(a) Find the quadratic approximation of $f(x, y)$ at $(x, y) = (-1, 1)$.

(b) Is $(-1, 1)$ a critical point for $f(x, y)$? Why or why not? If it is, what kind of critical point is it? Relative min, relative max, or saddle point?

5. (_____/10 points) Let $z = f(x,y)$, $x = u + v$ and $y = u - v$. Show that $\dfrac{\partial z}{\partial u} + \dfrac{\partial z}{\partial v} = 2\dfrac{\partial z}{\partial x}$.

6. (_____/9 points) Find the equation of the plane tangent to $xyz = -2$ at the point $(x, y, z) = (2, -1, 1)$.

7. (_____/5 points) 3 of the follow 4 statements are true. Circle the false statement.

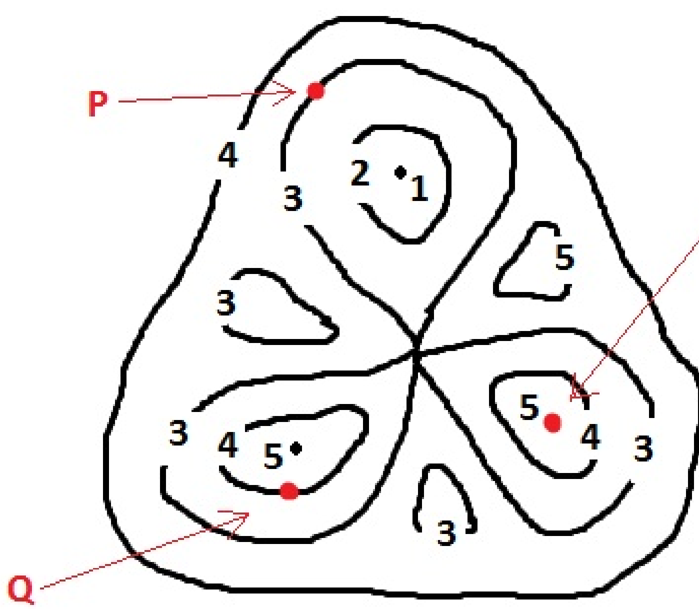
- If the first partials of $f(x,y)$ are continuous, then $f(x,y)$ is differentiable..
- If $f(x,y)$ is differentiable, then its first partials exist.
- If the first partials of $f(x,y)$ exist everywhere, then $f(x,y)$ is differentiable.
- If $f(x,y)$ is differentiable, then it is continuous.

8. (_____/15 points) Directional Derivative.

(a) Compute $D_{\mathbf{u}}f(0,1)$ where $f(x,y) = xy^2$ and $\mathbf{u}$ points in the same direction as the vector $\mathbf{v} = \langle 3, 4 \rangle$.

(b) What is the maximum value of the directional derivative of $f(x,y) = xy^2$ at $(x,y) = (0,1)$? Which direction maximizes the directional derivative?

(c) Given the following contour plot. Sketch the gradient vectors at the given points or mark the point with an "X" if the gradient should be the zero vector.



9. (_____/12 points) Use the method of Lagrange multipliers to find the maximum and minimum value of $f(x, y) = x + 2y$ constrained to the circle $x^2 + y^2 = 5$.

10. (_____/9 points) Approximate the integral $\iint_R x^2 + y\, dA$ where $R = [0, 4] \times [-2, 2]$ using 2×2 rectangles and the midpoint rule. Do **not** simplify your answer.

Math 2130-102 **Test #2** **October 24^{th}, 2011**

Name: __________________________________ **Be sure to show your work!**

If $F(x,y) = C$, then $\dfrac{dx}{dy} = -\dfrac{F_x}{F_y}$

If $F(x,y,z) = C$, then $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{F_y}{F_z}$

1. (____/9 points) Consider the surface $z = y - x^2$.

(a) Write down an equation for the level curves of this surface. What are these curves?

(b) Write down an equation for the trace made by intersecting this surface with the yz-plane.

(c) Make a rough sketch of this surface.

2. (____/10 points) The function $f(x,y) = \begin{cases} \frac{2x^2+y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ is not continuous.

Why? Explain your answer.

3. (_____/9 points) Suppose $z + y^2 + \sin(xyz) = 4$. Considering x and y as independent variables and z as a dependent variable, compute the implicit derivative $\dfrac{\partial z}{\partial x}$

4. (_____/10 points) Let $f(x, y) = x^3 + y^2 + xy$. Find the quadratic approximation of $f(x, y)$ at $(x, y) = (-1, 0)$.

5. (_____/10 points) Let $w = f(x, y, z)$, $x = g(u, v)$, $y = h(u, v)$, and $z = k(u, v)$. Write down the chain rule for $\frac{\partial w}{\partial u}$.

6. (_____/9 points) Find the equation of the plane tangent to $x^2 + 2y^2 + 3z^2 = 14$ at the point $(x, y, z) = (3, -1, 1)$.

7. (_____/9 points) Suppose $f(x, y)$ is a function with continuous second partials. In addition suppose that $f(x, y)$ has a critical point at $(-2, 5)$. Given the following data, state whether the second derivative test tells us if this critical point is a relative minimum, relative maximum, a saddle point, or if the test does not apply.

(a) $f_{xx}(-2, 5) = 2$, $f_{xy}(-2, 5) = 1$, and $f_{yy}(-2, 5) = 5$.

(b) $f_{xx}(-2, 5) = 2$, $f_{xy}(-2, 5) = 4$, and $f_{yy}(-2, 5) = 5$.

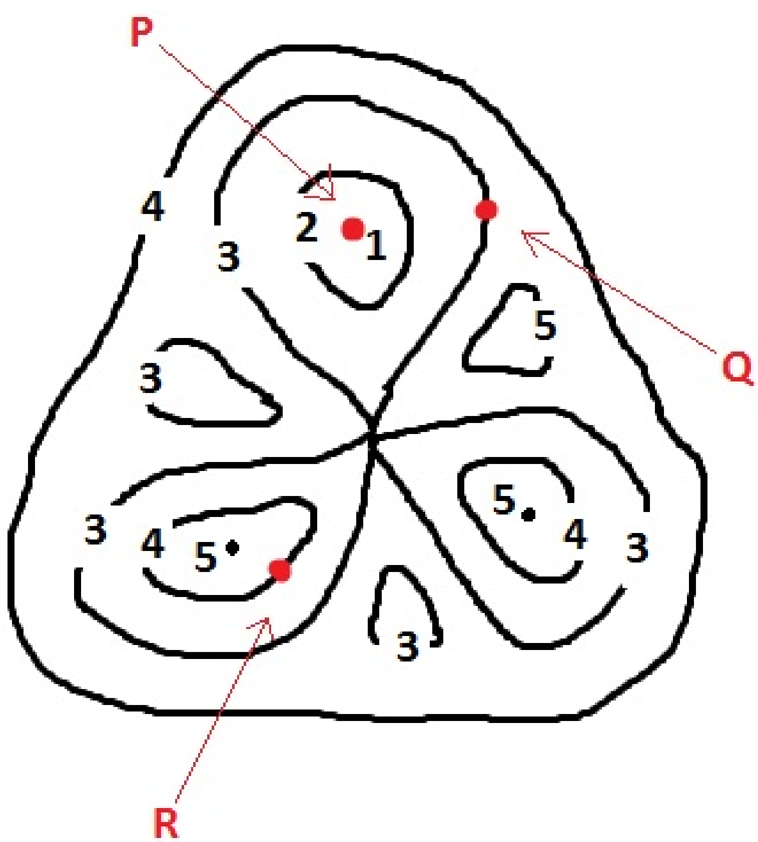
(c) $f_{xx}(-2, 5) = 9$, $f_{xy}(-2, 5) = 3$, and $f_{yy}(-2, 5) = 1$.

8. (_____/15 points) Directional Derivative.

(a) Compute $D_{\mathbf{u}}f(1,0)$ where $f(x,y) = x^2y^2 - 2x$ and $\mathbf{u}$ points in the same direction as the vector $\mathbf{v} = \langle 1, 2 \rangle$.

(b) What is the minimum value of the directional derivative of $f(x,y) = x^2y^2 - 2x$ at $(x,y) = (1,0)$? Which direction minimizes the directional derivative?

(c) Given the following contour plot. Sketch the gradient vectors at the given points or mark the point with an "X" if the gradient should be the zero vector.



9. (_____/10 points) Use the method of Lagrange multipliers to find the maximum and minimum value of $f(x, y) = xy$ constrained to the circle $x^2 + y^2 = 1$.

10. (_____/9 points) Approximate the integral $\iint_R xy^2 \, dA$ where $R = [-2, 2] \times [0, 8]$ using 2×2 rectangles and the midpoint rule. Do **not** simplify your answer.