Math 2130 # Test #3 **November 16^{th}, 2011**

Name: ________________________________ **Be sure to show your work!**

$$\begin{aligned} x &= \rho\cos(\theta)\sin(\varphi) \\ y &= \rho\sin(\theta)\sin(\varphi) \\ z &= \rho\cos(\varphi) \end{aligned}$$

$$J = \rho^2\sin(\varphi)$$

$$\cos^2(\theta) = \frac{1}{2}\left(1+\cos(2\theta)\right)$$

1. (____/15 points) Consider $\displaystyle\int_0^2 \int_{x/2}^1 e^{y^2}\,dy\,dx$.

   Sketch the region of integration and then **evaluate** the integral. *Hint:* $\int e^{y^2}\,dy$ is impossible to evaluate.

2. (____/15 points) Consider the integral $\displaystyle\iint_R \frac{3x-y}{x+y}\,dA$ where R is bounded by $y=3x+1$, $y=3x+2$, $y=-x$, and $y=-x+3$. Change coordinates using $u=3x-y$ and $v=x+y$. Do **not** evaluate the integral.

3. (_____/15 points) Consider the integral: $I = \int_0^3 \int_0^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-y^2}}^{0} x\, dz\, dy\, dx.$

(a) Rewrite I in the following order of integration: $\int\int\quad dy\, dx\, dz.$
Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates.
Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates.
Do **not** evaluate the integral.

4. (_____/15 points) Find the centroid of the **upper-half** of the unit ball: $x^2 + y^2 + z^2 \leq 1$ and $z \geq 0$.
Hint: Use symmetry to cut down the number of integrals you need to evaluate. The volume of a sphere of radius R is $\frac{4}{3}\pi R^3$

$$m = \iiint_E 1\, dV \qquad M_{yz} = \iiint_E x\, dv \qquad M_{xz} = \iiint_E y\, dv \qquad M_{xy} = \iiint_E z\, dv$$

5. (_____/14 points) Evaluate $\iiint\limits_E x^2 + y^2\, dV$ where E is the region bounded above by $z = 4 - x^2 - y^2$ and below by $z = 0$.

6. (_____/12 points) Compute the line integrals. **Note:** $ds = |\mathbf{r}'(t)|\, dt$ and $d\mathbf{r} = \mathbf{r}'(t)\, dt$

(a) $\displaystyle\int_C xy\, ds$ where C is the part of the circle $x^2 + y^2 = 4$ which lies in the first quadrant.

(b) $\displaystyle\int_C \mathbf{F} \bullet d\mathbf{r}$ where C is parametrized by $\mathbf{r}(t) = \langle 1, t, t^2 \rangle$, $0 \le t \le 1$, and $\mathbf{F}(x, y, z) = \langle xy, 5z^2, 1 \rangle$

7. (_____/14 points) For each of the following vector fields, $\mathbf{F}$:
Compute $\text{curl}(\mathbf{F}) = \nabla \times \mathbf{F}$ and $\text{div}(\mathbf{F}) = \nabla \bullet \mathbf{F}$. If $\mathbf{F}$ is conservative, find a potential function.

(a) $\mathbf{F}(x, y, z) = \langle 2x + y, x + z^2, 2yz \rangle$

Circle the correct answer: $\mathbf{F}(x, y, z)$ IS / IS NOT conservative.

(b) $\mathbf{F}(x, y, z) = \langle y^2 + z^2, x^2, 2xy \rangle$

Circle the correct answer: $\mathbf{F}(x, y, z)$ IS / IS NOT conservative.