Math 2130

Name:

Be sure to show your work!

$$\begin{array}{rcl} x & = & \rho\cos(\theta)\sin(\varphi) & J = \rho^2\sin(\varphi) \\ y & = & \rho\sin(\theta)\sin(\varphi) \\ z & = & \rho\cos(\varphi) & \cos^2(\theta) = \frac{1}{2}\left(1 + \cos(2\theta)\right) \end{array}$$

1. (____/15 points) Consider
$$\int_0^2 \int_{x/2}^1 e^{y^2} dy dx$$
.

Sketch the region of integration and then **evaluate** the integral. *Hint:* $\int e^{y^2} dy$ is impossible to evaluate.

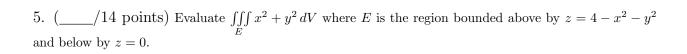
2. (____/15 points) Consider the integral
$$\iint_R \frac{3x-y}{x+y} dA$$
 where R is bounded by $y=3x+1$, $y=3x+2$, $y=-x$, and $y=-x+3$. Change coordinates using $u=3x-y$ and $v=x+y$. Do **not** evaluate the integral.

- 3. (____/15 points) Consider the integral: $I = \int_0^3 \int_0^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-y^2}}^0 x \, dz \, dy \, dx$.
- (a) Rewrite I in the following order of integration: $\iint dy dx dz$. Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates. Do **not** evaluate the integral.

- (c) Rewrite I in terms of spherical coordinates. Do **not** evaluate the integral.
- 4. (____/15 points) Find the centroid of the upper-half of the unit ball: $x^2 + y^2 + z^2 \le 1$ and $z \ge 0$. Hint: Use symmetry to cut down the number of integrals you need to evaluate. The volume of a sphere of radius R is $\frac{4}{3}\pi R^3$

$$m = \iiint_E 1 \, dV$$
 $M_{yz} = \iiint_E x \, dv$ $M_{xz} = \iiint_E y \, dv$ $M_{xy} = \iiint_E z \, dv$



6. (____/12 points) Compute the line integrals.

Note:
$$ds = |\mathbf{r}'(t)| dt$$
 and $d\mathbf{r} = \mathbf{r}'(t) dt$

(a) $\int_C xy \, ds$

where C is the part of the circle $x^2 + y^2 = 4$ which lies in the first quadrant.

(b) $\int_C \mathbf{F} \cdot d\mathbf{r}$

where C is parametrized by $\mathbf{r}(t) = \langle 1, t, t^2 \rangle$, $0 \le t \le 1$, and $\mathbf{F}(x, y, z) = \langle xy, 5z^2, 1 \rangle$

- 7. (____/14 points) For each of the following vector fields, **F**: Compute $\operatorname{curl}(\mathbf{F}) = \nabla \times \mathbf{F}$ and $\operatorname{div}(\mathbf{F}) = \nabla \bullet \mathbf{F}$. If **F** is conservative, find a potential function.
- (a) $\mathbf{F}(x, y, z) = \langle 2x + y, x + z^2, 2yz \rangle$

Circle the correct answer: $\mathbf{F}(x, y, z)$ IS / IS NOT conservative.

(b) $\mathbf{F}(x, y, z) = \langle y^2 + z^2, x^2, 2xy \rangle$