Math 2130-101A # Test #1 **September 21^{st}, 2012**

Name: ______________________________

Be sure to show your work!

$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \dfrac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v}$

$\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$

$\mathbf{r}''(t) = \left(\dfrac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t)$

$\kappa = \dfrac{|f''(x)|}{\left(1 + (f'(x))^2\right)^{\frac{3}{2}}}$

1. (____/17 points) Let $\mathbf{u} = \langle 1, 2, 3 \rangle$, $\mathbf{v} = \langle -1, 0, 1 \rangle$, and $\mathbf{w} = \langle 2, 1, 1 \rangle$.

(a) Find the area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{w}$.

(b) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trigonometric functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(c) Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

2. (____/20 points) Lines

(a) Let ℓ_1 be the line parametrized by $\mathbf{r}_1(t) = \langle 1+t, 2-2t, -t\rangle$ and ℓ_2 be the line parametrized by $\mathbf{r}_2(t) = \langle 3-2t, 4t, -1+2t\rangle$. Determine if ℓ_1 and ℓ_2 are the same, parallel, intersecting, or skew.

(b) Parametrize the line **segment** through $P = (1, 2, -1)$ and $Q = (3, 2, 1)$. Remember to specify bounds for your parameter: $??? \leq t \leq ???$.

(c) Find a parametrization for the line tangent to $\mathbf{r}(t) = \langle t, t^2, t^3\rangle$ at $t = -1$.

3. (____/14 points) Find a (scalar) equation for the plane which contains the line $\mathbf{r}(t) = \langle 1, -2, 1\rangle + \langle -1, 0, 2\rangle t$ and the point $P = (-3, 2, -1)$.

4. (____/16 points) Let $\mathbf{r}(t) = \langle e^{-t}, t, \sin(t)\rangle$ where $-\pi \leq t \leq 6\pi$.

(a) Set up an integral which computes the arc length of the curve parametrized by $\mathbf{r}(t)$. Do **not** attempt to evaluate this integral.

(b) Find the curvature of $\mathbf{r}(t)$. Do **not** worry about simplifying.

5. (____/17 points) Consider the curve $\mathbf{r}(t) = \langle 5\sin(t), 3, 5\cos(t)\rangle$.

(a) Find the **TNB**-frame for $\mathbf{r}(t)$.

(b) *Note:* The curve parametrized by $\mathbf{r}(t)$ lies in a plane.
Find the scalar equation for the plane containing this curve.

In general, what guarantees that we have a planar curve? ______________________________

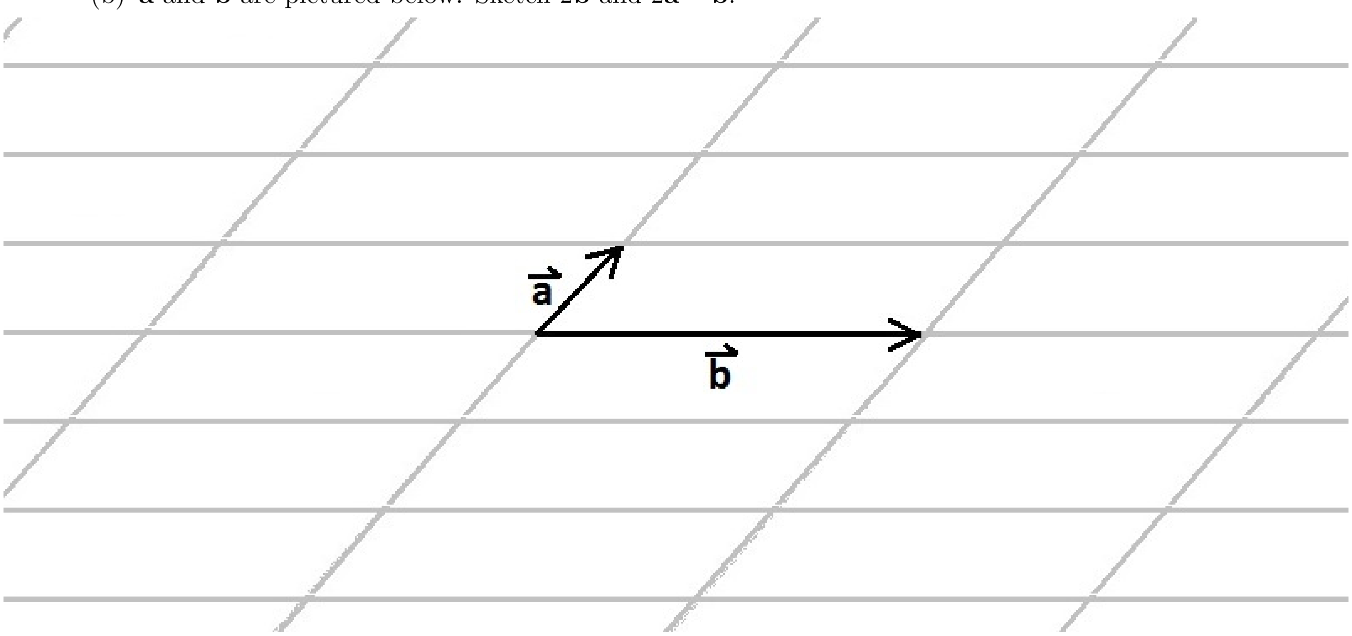
6. (____/16 points) No numbers here.

(a) Choose **ONE** of the following:

I. Let $\mathbf{v}$ and $\mathbf{w}$ be **unit** vectors. Show that $|\mathbf{v} \times \mathbf{w}|^2 + (\mathbf{v} \bullet \mathbf{w})^2 = 1$.

II. Suppose that $|\mathbf{r}(t)| = c$ (for some constant c). Show that $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are orthogonal.

(b) $\mathbf{a}$ and $\mathbf{b}$ are pictured below. Sketch $2\mathbf{b}$ and $2\mathbf{a} - \mathbf{b}$.



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Name: ____________________________________ **Be sure to show your work!**

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v}$$

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$\mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t)$$

$$\kappa = \frac{|f''(x)|}{\left(1 + (f'(x))^2\right)^{\frac{3}{2}}}$$

1. (____/16 points) Let $\mathbf{u} = \langle 1, 2, -1\rangle$, $\mathbf{v} = \langle 0, 1, -2\rangle$, and $\mathbf{w} = \langle 2, -1, -1\rangle$.

(a) Find all possible **unit** vectors that are parallel to $\mathbf{u}$.

(b) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trigonometric functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(c) Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

2. (____/17 points) Lines

(a) Let ℓ_1 be the line parametrized by $\mathbf{r}_1(t) = \langle 1 + 2t, 3 - 2t, -1 - t \rangle$ and ℓ_2 be the line parametrized by $\mathbf{r}_2(t) = \langle t, 1 + 2t, 2 - 2t \rangle$. Determine if ℓ_1 and ℓ_2 are the same, parallel, intersecting, or skew.

(b) Parametrize the line which passes through the point $P = (2, 3, -1)$ and is parallel to the line parametrized by $\mathbf{r}(t) = \langle 1 - 2t, 3 + 4t, 6 - 5t \rangle$,

(c) Find a parametrization for the line tangent to $\mathbf{r}(t) = \langle 3t + 1, t^2, e^t \rangle$ at $t = 0$.

3. (_____/13 points) Let $P = (1, 0, -1)$, $Q = (2, 1, 3)$, and $R = (3, 2, 1)$.

(a) Find a (scalar) equation of the plane which contains the points P, Q, and R.

(b) Find the area of ΔPQR (the triangle with vertices P, Q, and R).

4. (_____/12 points) Set up the integral which computes the arc length of the curve parametrized by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ where $-2 \leq t \leq 5$. Do **not** try to evaluate this integral.

5. (____/14 points) Curvature

(a) Find the curvature of $\mathbf{r}(t) = \langle t^3, t, \sin(t) \rangle$. Do **not** worry about simplifying.

(b) Find the curvature of $y = e^{2t}$.

6. (____/14 points) A Helix Problem.

(a) Find the **TNB**-frame for $\mathbf{r}(t) = \langle \cos(t), \sin(t), 2t \rangle$.

(b) Obviously this helix (or any other helix) does not lie in a plane.
What about the **TNB**-frame shows this is the case?

7. (____/14 points) No numbers here.

(a) Choose **ONE** of the following:

I. Let $\mathbf{p} = \text{proj}_{\mathbf{w}}(\underline{v})$ and $\mathbf{q} = \mathbf{v} - \mathbf{p}$. Show $\mathbf{p}$ and $\mathbf{q}$ are orthogonal.

II. Use properties of the derivative operator to compute $\frac{d}{dt}\left[\mathbf{r} \bullet (\mathbf{r}' \times \mathbf{r}'')\right]$ and simplify.

(b) $\mathbf{a}$ and $\mathbf{b}$ are pictured below. Sketch $-2\mathbf{a}$ and $\mathbf{a} - \mathbf{b}$.